PHYSICS DEPARTMENT

RUTGERS
THE STATE UNIVERSITY
OF NEW JERSEY

MICROSCOPIC PHENOMENA AND A MODERN APPROACH TO TURBULENCE

J. A. Johnson III, S. Chen, L. I,
W. Jones, R. Ramaiah, J. Santiago

MICROSCOPIC PHENOMENA AND A MODERN

APPROACH TO TURBULENCE

Joseph A. Johnson III, Shen Chen, Lin I, Walter Jones,
Raghu Ramaiah, Jaime Santiago

Department of Physics
Rutgers University, New Brunswick, N. J. 08903

April 2, 1979

Abstract

This document is a final technical report on a National Aeronautics and Space Administration research grant titled "Three Body Recombination in Supersonic Flow", NSG 1314, June, 1976 through July, 1978, from NASA Langley Research Center. The investigators proposed to look for diagnostic techniques and experimental evidence to support new approaches to the turbulence problem using the shock tube and to undertake related theoretical studies as well. Evidence to support a kinetic theory of turbulence has been found. An overview of these and other achievements to date is given herein.

Table of Contents

Table Captions

Table I   The Performance of Pressure Loaded Arc Driven   54
            Shock Tubes. See text for the explanation of
            symbols.

Figure Captions

Figure 1 The growth of the turbulent intensity of bursts 13

in an unstable detonation front. The Reynolds

number is based on the displacement thickness as

measured from an interferogram of the front from

reference 9. The quantity $(\overline{\Delta S^2})^{1/2}$ is the root

mean squared averaged value of the fluctuations

about an average fringe shift in the turbulent

burst. The quantity I is defined in the text.

The dashed straight line is a fit to the four

data points indicated which shows a slope

$(d(\log I)/dRe = 3.2 \times 10^{-3})$ consistent with

theoretical expectations.

Figure 2 Overall Shock Tube Flow. This is an (x,t) 16

diagram. A partition separates two regions in

the shock tube with high pressure gas occupying

$x < o$ at $t < o$ and the gas originally at low

pressure occupying $x > o$ at $t < o$. Examples

of the various discontinuities and wave phenomena

in the shock tube subsequent $(t>o)$ to the rupture

of the partition at $x = o$, $t = o$ are shown.

(S) is the shock wave; (C) is the contact surface;

(T) is the tail of the expansion fan (in this

case $(v_3-a_3) > 0$); (H) is the head of the expan-

sion fan. (See text.)

(v)

the shock wave and D shows the output of the interferometer with the amplitude modulation resulting from fringe shifts. A sample of the data, with phototube output (arbitrary units) versus time (50 μsec/div.), is displayed as an insert. $T_{c,c}$ is the calculated value of the interval, using the shock wave's measured Mach number, M_s, and assuming ideal gas flow, between the arrival of the shock wave and the arrival of the contact surface at the same location.

(b) $M_s = 2.2$, $(Re^t/cm) = 4.4 \times 10^5$; (c) $M_s = 2.0$, $(Re^t/cm) = 2.7 \times 10^5$; (d) $M_s = 2.0$, $(re^t/cm) = 3.8 \times 10^5$.

$w = 1661 \, Z - 1405$ in which $w = tD^{-1} \times 10^{-3} \, sec^{-1}$
and $Z = p_B/p_L$. The data in (b) are taken from
the β_1 fluctuation (see text).

I. Introduction

We have been motivated to investigate alternatives to the phenomenological approach to turbulence by the well-known deficiencies in that approach. These deficiencies include: (1) the use of empirical constants not derivable from the fundamental constants of nature; (2) the inability to consistently explain the existence and systematics of large coherent structures in "steady" turbulence; (3) the inadequate characterizations of transport effects observed in reacting transitional flow, especially, flow with exothermic processes; (4) the incomplete statistical description of the randomly fluctuating gasdynamical variables. Generally stated, we have sought to create and understand turbulent environments where phenomena might be observed which would allow us to distinguish and evaluate new approaches to turbulence. In parallel with these efforts, we have sought to extend the new approaches into regimes which are accessible to our measurement capabilities. This report reviews the principal specific results within that general context from our beginning two year studies of turbulence with NASA support.

II. Reaction Rate Distortion in Turbulent Flow

Consider the irreversible chemical reaction A + B $\rightarrow$ P in a turbulent gas from a classical point of view. Impact parameters between particles ℓ and k are $b_{\ell k}$, the separation between their centers, and ε, the angle between their original and final velocities in the center of mass. Each particle has a characteristic mass m and a characteristic internal energy E_ℓ. Let's assume that: (1) interactions are unimportant; (3) only binary collisions need be considered. Mass and total momentum are to be strictly conserved in all collisions. Energy is also conserved; however, the product P is produced in an excited state (with excess internal energy E_{p*}) from which it very promptly relaxes to its characteristic internal state E_p through some unspecified noncollisional (radiative) process. We shall apply the approach used by Tsugé[1] to this problem as we look for an explicit influence of turbulence on the apparent reaction rate.

Using the Klimontovich N-particle distribution function[2] with the six dimensional phase space coordinate Z, $\overset{\circ}{f}(Z,t) \cong \sum\limits_{s=1}^{N} \delta[Z-Z^{(s)}(t)]$, the Master Boltzmann Equation for A may be written as:

$$(\frac{\partial}{\partial t} = v_A^r \frac{\partial}{\partial x^r})\overset{\circ}{f}_A = \sum_\ell \{\int (\overset{\circ}{f}_A'\overset{\circ\wedge}{f}{}' - \overset{\circ}{f}_A\overset{\circ\wedge}{f}_\ell) |\vec{v}_A - \overset{\wedge}{\vec{v}}_\ell| b_{A\ell} db_{A\ell} d\varepsilon d^3\hat{v}_\ell\}$$

$$- \int (F) \overset{\circ}{f}_A'\overset{\circ}{f}_B' |\vec{v}_A - \overset{\wedge}{\vec{v}}_B| b_{AB} db_{AB} d\varepsilon d^3\hat{v}_B \qquad \text{II-(1)}$$

where $\ell = A, B$, then P and the function "F" is the fraction of collisions between A and B which result in formation of P. The equation for B is the same as II-(1) with the B label replacing A:

$$\left(\frac{\partial}{\partial t} + v_B^r \frac{\partial}{\partial x^r}\right)\overset{o}{f}_B = \underset{\ell}{\Sigma}\{\ (B{\to}A\ above)\ \}- \int (F)\overset{o}{f'_B}\overset{\wedge}{f'_A}|\vec{v}_B - \overset{\rightarrow}{\hat{v}}_A|\ b_{AB}db_{AB}d\epsilon d^3\hat{v}_A$$

$$\text{II-}(2)$$

For the product P:

$$\left(\frac{\partial}{\partial t} + v_P^r \frac{\partial}{\partial x^r}\right)\overset{o}{f}_P = \Sigma\{\int\ (\overset{o}{f'_P}\overset{\overset{o}{\wedge}}{f'_\ell} - \overset{o}{f}_P\overset{\overset{o}{\wedge}}{f}_\ell)|\vec{v}_P - \overset{\rightarrow}{\hat{v}}_\ell|b_{P\ell}db_{P\ell}d\epsilon d^3\hat{v}_\ell\}$$

$$+ \int (F)\overset{o}{f}_A\overset{\overset{o}{\wedge}}{f}_B|\vec{v} - \overset{\rightarrow}{\hat{v}}_B|\ \partial\ (m_A\vec{v}_A + m_B\vec{v}_B - m_P\overset{\rightarrow}{\hat{v}}_P)b_{AB}db_{AB}d\epsilon d^3v_A d^3\hat{v}_B$$

$$\text{II-}(3)$$

Equation II-(1) simply says that the rate of change in the number of molecules in a phase-space volume element $d^3x d^3v_A$ is equal to the sum of the differences of the rate of molecule A into and out of the specified volume element due to collisions between: A and A; A and B; A and P; less the rate resulting from the formation of P. Equation II-(2) has the same meaning for molecule B. Equation II-(3) is different since there are no collisions involving P which can cause the reaction; however, a fraction of the collisions of A and B do produce P. The inclusion of a delta function is to insure that only those collisions which conserve momentum are allowed and to guarantee

that the resulting new P molecules will be in the specified volume element $d^3x d^3v_p$ around $\vec{x}$ and $\vec{v}_p$.

Following Tsugé, we can define: $\overline{\overset{o}{f}} = f$; $\overline{\overset{o}{f_A}\overset{\wedge}{f_B}} = f_{AB}(Z_A, \hat{Z}_B)$

$$\overline{\overset{o}{f_A}\overset{\wedge}{f_A}} = \overline{\sum_{s=1}^{N} \delta[Z_A - Z_A^{(s)}(t)] \sum_{\hat{s}=1}^{N} \delta[Z_A - Z_A^{(\hat{s})}(t)]} = \overline{\sum_{s \neq \hat{s}} \delta[Z_A - Z_A^{(s)}(t)] \delta[Z_A - Z_A^{(\hat{s})}(t)]}$$

$$+ \delta(Z_A - \hat{Z}_A) \overline{\sum_{s=1}^{N} \delta[Z_A - Z_A^{(s)}(t)]} = f_{AA}(Z_A, \hat{Z}_A) + \delta(Z_A - \hat{Z}_A) f_A(Z).$$

Turbulent-like effects are obtained without ad hoc assumptions from situations where, defining $\psi(Z, \hat{A}) \equiv f^{II}(Z, \hat{Z}) - f(Z) f(\hat{Z})$, one finds a history with $\psi(Z, Z) \neq 0$. The importance of this approach to turbulent flow without reactions has already been discussed.[3] By taking the averages of equations II-(1), II-(2), and II-(3) and then, in turn, multiplying each equation by a moment function $\alpha(\vec{v}_\ell) = \alpha_\ell$ and integrating over d^3v_ℓ, we can add the three resulting equations and find (using standard notation):

$$\sum_\ell \left\{ \frac{\partial}{\partial t} \int \alpha_\ell f_\ell d^3v_\ell + \frac{\partial}{\partial x^r} \int \alpha_\ell v_\ell^r f_\ell d^3v_\ell - \int \left(\frac{\partial \alpha_\ell}{\partial t} + v_\ell^r \frac{\partial \alpha_\ell}{\partial x^r} \right) f_\ell d^3v_\ell \right.$$

$$= \sum_{k,\ell} \left\{ \frac{1}{2} \int (\alpha_\ell' + \alpha_k' - \alpha_\ell - \alpha_k) [f_\ell \hat{f}_k + \psi_{\ell k}(Z_\ell, \hat{Z}_k)] d^3v_\ell d^3v_k d^3K_{\ell k} \right\}$$

$$+ \int \alpha_P(F) [f_A \hat{f}_B + \psi_{AB}(Z_A, \hat{Z}_B)] \delta(m_A \vec{v}_A + m_B \vec{\hat{v}}_B - m_P \vec{v}_P) d^3v_A d^3\hat{v}_B d^3v_P d^3K_{AB}$$

$$- \int (\alpha_A + \alpha_B)(F) [f_A \hat{f}_B + \psi_{AB}(Z_A, \hat{Z}_B)] d^3v_A d^3\hat{v}_B d^3K_{AB}$$

$$\text{II-(4)}$$

where k also stands for A, B, or P, and $d^3K_{AB} = |\vec{v}_A - \vec{v}_B| b_{AB} db_{AB} d\varepsilon$.

Now we can define, in the usual manner and again using standard notation, the hydrodynamical quantities of interest:[3]

$$n_\ell = \int f_\ell d^3 v_\ell, \quad \rho = \sum_\ell m_\ell n_\ell, \quad \vec{u}_\ell = \frac{1}{n_\ell} \int \vec{v}_\ell f_\ell d^3 v_\ell, \quad \vec{u} = \frac{1}{\rho} \sum_\ell n_\ell m_\ell \vec{u}_\ell;$$

$$\vec{\xi}_\ell \equiv \vec{u}_\ell - \vec{u} = \frac{1}{n_\ell} \int (\vec{v}_\ell - \vec{u}) f_\ell d^3 v_\ell = \frac{1}{n_\ell} \int \vec{c}_\ell f_\ell d^3 v_\ell, \vec{c}_\ell \equiv \vec{v}_\ell - \vec{u};$$

$$n = \sum_\ell n_\ell, \quad E = (\sum_\ell n_\ell E_\ell)/n, \quad 3KT/2 = (\sum_\ell m_\ell \int c_\ell^2 f_\ell d^3 v_\ell)/2n;$$

$$P_{ij}^\ell \equiv m_\ell \int c_\ell^i c_\ell^j f_\ell d^3 v_\ell, \quad P_{ij} = \sum_\ell P_{ij}^\ell; \text{ and } q_i = \sum_\ell \int c_\ell^i (\frac{m_\ell c_\ell^2}{2} + E_\ell) f_\ell d^e v_\ell.$$

When $\alpha_\ell = m_\ell$ and $\alpha_\ell^i = m_\ell v_\ell^i$ Equation II-(4) becomes, using the definitions above:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x^r} (\rho u^r) = 0 \quad \text{and} \quad \frac{\partial (\rho u^i)}{\partial t} + \frac{\partial}{\partial x^r} [P_{ir} + \rho u^i u^r] = 0$$

respectively since mass and momentum are strictly conserved. Thus, these conservation equations are unchanged from their usual appearance even for inelastic collisions. When $\alpha_\ell = \frac{1}{2} m_\ell v_\ell^2 + E_\ell$, the summation term on the right hand side of equation (4) is zero due to strict conservation of energy in the elastic collisions. The remaining terms on the right hand side are:

$$\int [-\frac{(m_A \vec{v}_A + m_B \vec{v}_B)^2}{2m_p} + E_p] (F) [f_A \hat{f}_B + \psi_{AB}(Z_A, \hat{Z}_B)] d^3 v_A d^3 \hat{v}_B d^3 K_{AB}$$

$$- \int (\frac{m_A v_A^2 + m_B \hat{v}_B^2}{2} + E_A + E_B) (F) [f_A \hat{f}_B + \psi_{AB}(Z_A, \hat{Z}_B)] d^3 v_A d^3 \hat{v}_B d^3 K_{AB}$$

$$= \int [E_p - E_A - E_B - \frac{1}{2} \mu V^2] (F) [f_A \hat{f}_B + \psi_{AB}(Z_A, \hat{Z}_B)] d^3 v_A d^3 \hat{v}_B d^3 K_{AB}$$

where $\mu = m_A m_B / (m_A + m_B)$ and $V = |\vec{v}_A - \vec{v}_B|$. Evaluating the left hand side of the equation (4), we achieve

$$\frac{\partial}{\partial t} [\frac{3}{2} nkT + \frac{1}{2} \rho u^2 + nE] + \frac{\partial}{\partial x^r} [\frac{1}{2} \rho u^2 u^r + \frac{3}{2} nkT u^r + P_{ir} u^i + q_r + nE u^r]$$

$$= \int [E_p - E_A - E_B - \frac{1}{2} \mu V^2] (F) [f_A \hat{f}_B + \psi_{AB}(Z_A, \hat{Z}_B)] d^3 v_A d^3 \hat{v}_B d^3 K_{AB} \qquad \text{II-(5)}$$

Of course, the reaction is prohibited when $E_p - E_A - E_B > \frac{1}{2} \mu V^2$. If $E_p > E_A + E_B$ but $E_p - E_A - E_B < \frac{1}{2} \mu V^2$, then part of $\frac{1}{2} \mu V^2$ is used to increase the internal energy of the system and the remainder is released in the form of heat. If $E_p < E_A + E_B$, then heat is produced in the amount $(E_A + E_B + \frac{1}{2} \mu V^2 - E_p)$ with each reaction. Equation II-(5) is consistent with these expectations since the energy dependence is contained in "F".

Our new result is contained in the term with a contribution from $\psi_{AB}(Z_A, \hat{Z}_B)$. Equation II-(5) asserts that, when the reaction is allowed $(F \neq 0)$, it can be affected by turbulence in a direct manner which is distinguishable from that arising from ordinary turbulent diffusion. A new apparent reaction rate fraction, F_{App}, can be defined:

$$F_{App} \equiv \frac{(F)[f_A \hat{f}_B + \psi_{AB}(z_A, \hat{z}_B)]}{f_A \hat{f}_B} \quad , \quad \text{and} \quad \frac{F_{App} - F}{F} = \frac{\psi_{AB}(z_A, \hat{z}_B)}{f_A \hat{f}_B}$$

Thus, there is an explicit influence of turbulence on the rate of production of P; the reaction is distorted in all cases where the condition ($\psi_{AB}/f_A \hat{f}_B \ll 1$) is violated.

III. Turbulent Bursts in a Shock Tube

Turbulent discontinuities in shock tubes are usually thought to be boundary layer-like insofar as transition to turbulence is concerned.[4] However, theoretical treatments which rely on the techniques from linear stability formulations have seemed to be inadequate for the analysis of instabilities in contact surfaces[5] and detonation fronts.[6] Tsugé's[7] treatment, by contrast, is not restricted to the small perturbation regime; macroscopic turbulent effects are shown to evolve from a kinetic theory with reduced microscopic (molecular) chaos. Furthermore, transition related turbulent bursts in a Blasius boundary layer experiment are successfully interpreted.[7] Here in this paper, we suggest an extension of Tsugé's approach to a wider class of unstable boundary layers, viz., to analogous unstable gasdynamic discontinuities in shock tubes.

When one focuses on those fluctuations which experience maximum amplification and identifies these fluctuations as the experimentally observed fluctuations, the theoretical solutions dictate the familiar form

$$I = \exp[\phi(Re,\lambda)]. \qquad \text{III-(1)}$$

Solutions of equation III-(2) are well known.[7,8,9] The Reynolds number has its usual meaning. The expansion parameter λ has the units of frequency. The quantity I is an explicit function of turbulent intensity, $I \equiv Q/Q_o$, where Q is the observed turbulent

intensity and Q_o is a minimum intensity level which must be exceeded in order for fluctuations to grow. The function ϕ has no empirical constants; all constants are derivable from boundary conditions. Therefore, this formulation is non-phenomenological and has unique physical consequences.[10,11]

If one considers the solutions of equation II-(1) in the form of a family of curves of I' vs Re (where I' = log I) at selected values of the nondimensional ratio $\lambda\nu/u_d^2$ (ν is the kinematic viscosity and u_d is a characteristic free stream boundary layer velocity), systematic behaviors emerge. The curves show: a first regime with $(dI'/dRe) > 0$, $(d^2I'/dRe^2) > 0$; a middle regime which is essentially linear, with $(dI'/dRe) > 0$, $(d^2I'/dRe^2) = 0$; and a third regime with $(dI'/dRe) > 0$ initially and $(d^2I'/dRe^2) < 0$. The linear middle regime has a slope which we shall name m and an intercept on the Re axis which we name Re_n. Extrapolated to I' = 0, the middle regime gives, at Re = Re_n, $Q = Q_o$. The third regime has a maximum in I' of value I'_{mx} at a Reynolds number which we now name Re_p.

The extension of this kinetic theory of turbulence to any given set of experimental data on unstable boundary layer flow with turbulent bursts is therefore straightforward. If there is a linear regime in a plot of log $Q_{(exp)}$ vs Re, then the slope of the line should be well estimated by a m appropriate for the Reynolds number range being treated. If there is a maximum in the $Q'_{(exp)}$ (= log $Q_{(exp)}$ vs Re trajectory, then the value of Re at $Q'_{(exp),mx}$ is a measure of Re_p. One obtains from the measure of Re_p an estimate of λ (from $\lambda\nu/u_d^2$), which

can be called the characteristic frequency of the instability, and in turn an estimate of Re_n. Since in the linear regime the experimental data will fit $\log Q = b + (m)(Re)$, the estimate of Re_n gives an estimate of Q_o by

$$\log Q \Big|_{Re=Re_n} = \log Q_o = b + m \cdot Re_n \quad . \qquad \text{III-(2)}$$

For example, for Shubauer's experiment (as reported in reference 1) the solutions to equation III-(1) in the appropriate Reynolds number range show: a slope m in the second regime independent of λ, i.e., $m \neq m(\lambda)$ and $m \equiv (dI'/dRe) = (3.3\pm0.2) \times 10^{-3}$; a systematic dependence of the characteristic wavelength on Re_p from the third regime which is given by $(\lambda\nu/u_d^2) = (4.6\pm0.3) \times 10^{-5} \times \exp[-(0.34\pm0.02) \times 10^{-3}Re_p]$; a correlation between Re_n and Re_p which is found to be $Re_n = (0.29\pm0.06) + (0.44\pm0.02)Re_p$. In the data with turbulent bursts, a linear regime is found with slope $m_{(exp)} = dQ'/dRe = 3.5 \times 10^{-3}$ and a Q'-axis intercept $b_{(exp)} = -11.45$. A maximum in the Q' vs Re plane occurs at $Re_p = 3.19 \times 10^3$; this gives $(\lambda\nu/u_d^2) = 1.55 \times 10^{-5}$ from the expression above in agreement with the results in reference 1. In addition, using the Q' axis intercept and the relationship between Re_n and Re_p (which provides $Re_n = 1.4 \times 10^3$) given above, our estimate of Q_o from equation III-(2) is $Q_o = 1.5 \times 10^{-3}$. This corresponds to $Q_{mx}/Q_o = 120$, i.e., a required minimum turbulent intensity level which is 120 times smaller than the observed maximum intensity in the turbulent bursts.

Consider next a curved unstable detonation wave moving in a shock tube with constant velocity u_o. The non-one dimensional nature of this discontinuity is assumed (as is now conventional) to have resulted from the shear induced by the curvature related dynamics. Let's model the turbulent front by an apparent boundary layer which is transverse to and symmetric about the axis of propagation. That is, its smallest dimension is at the center of the tube; its largest dimensions (small with respect to the radius of curvature of the front) are at the shock tube's walls. This apparent boundary layer is perpendicular to the "normal"boundary layer at the shock tube's walls. The local Reynolds number in it can be defined using a measured local characteristic length, an overall characteristic kinematic viscosity, and an overall apparent characteristic "free stream" transverse boundary layer velocity $u_d \cong u_o$ $(\alpha^2/8)$ arising from the curvature where α is the total angle subtended by the curved front.

White's turbulent detonation wave experiments[12] are amenable to this extension. Measurements of turbulent fluctuation levels and local characteristic lengths are obtained from fringe shift fluctuations in his interferograms using well-known procedures.[13] Specifically, for a self-sustaining detonation into $2H_2 + O_2 + 2CO$ with $u_o = 2.19$ km/s and $\alpha = 0.047$ rad, turbulent bursts are observed for which $m_{(exp)} = 3.2 \times 10^{-3}$ and $Re_p = 208$. This provides $(\lambda v/u_d^2) = 4.3 \times 10^{-5}$. In addition, the fit to the linear regime and the calculated re_n (= 92) give a ratio of maximum observed intensity to minimum required intensity $Q_{mx}/Q_o = 2$. Figure 1 shows these results.

Thus might a wide variety of discontinuities with similar nonlinear instabilities be treated. We notice that, in the two instances above, the values of Q_{mx}/Q_o are quite different. This implies radically different turbulence producing mechanisms, an implication which is qualitatively consistent with our intuition. However, it remains to explore similarly obtained values of Q_{mx}/Q_o and $(\lambda\nu/u_d^2)$ for other unstable systems before new quantitative insight can be derived.
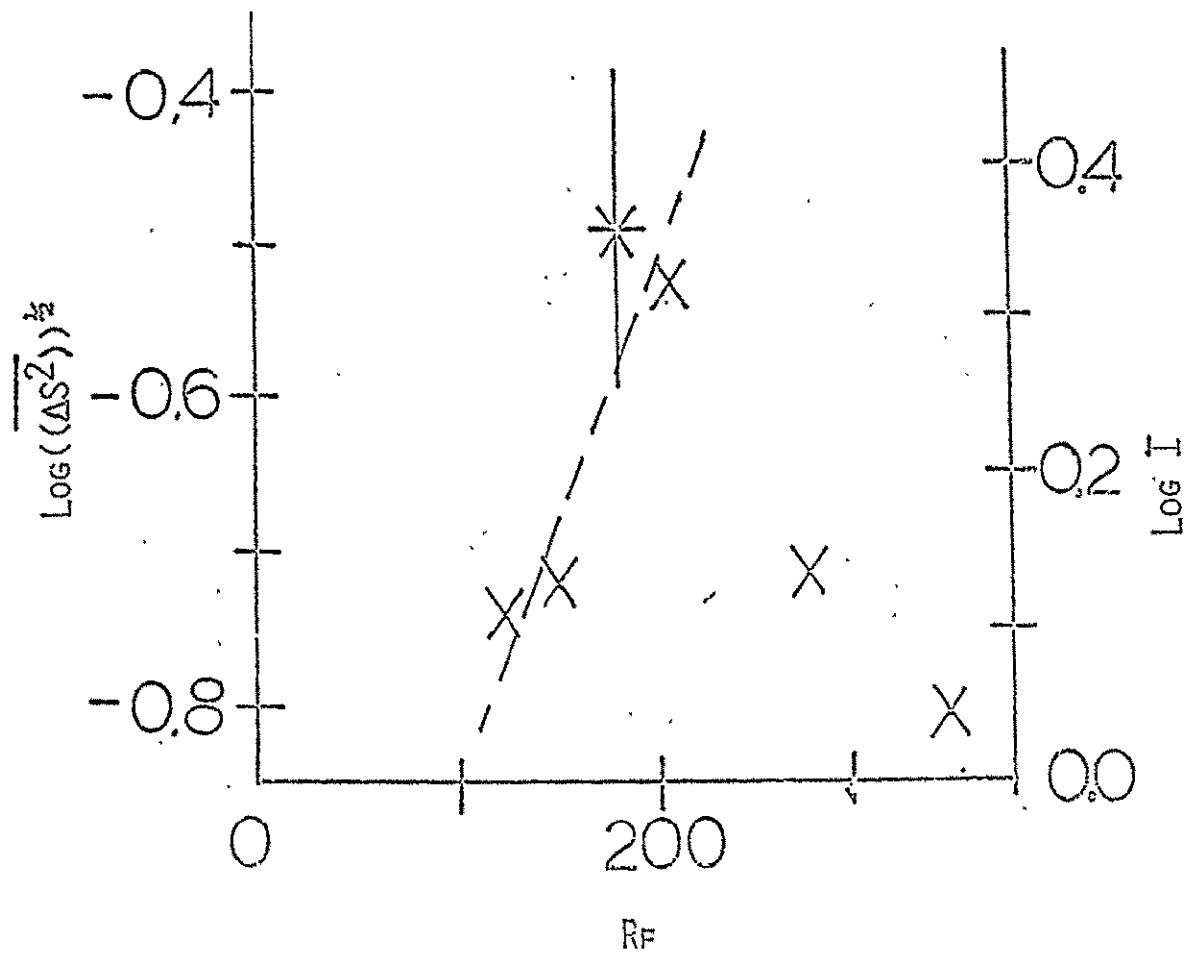
1.

IV.   Driver Gas Flow with Fluctuations

A.   Introduction

The conventional Navier Stokes equations do not adequately describe turbulent flow. None of the phenomenological approaches which have successfully characterized some turbulent systems seems derivable from a molecular view of matter. On the other hand, a recent series of papers[14,15,16,17] suggests an alternative non-empirical view of turbulence which is based on a modification of the usual view of molecular chaos. A set of modified Navier Stokes equations is derived as well as a variety of unique physical consequences.

Among these is a characterization of transition to turbulence in boundary layer flow without the usual restriction to linear instabilities. Contact surfaces are ordinarly thought to be boundary-layer like insofar as transition to turbulence is concerned.[18] Linear approaches seem to have failed in many important cases.[19] Therefore, we have explored the potential usefulness of contact surface instabilities in driver gas flow in a shock tube for new insight on turbulence, paying special attention to the possible applicability of the new approach from kinetic theory to our observations.

B.   Background: Shock Tube Flow

For idealized shock tube calculations, infinitely fast rupture for the diaphragm material separating the high pressure and low pressure sections is usually assumed. (See, for example, Thompson[20] on one dimensional unsteady flow.) This provides the

well-known flow regions indicated in the (x,t) diagram in
Figure 2: a region (1) of un-shocked, driven gas at the original
driven gas conditions (p_1, T_1, ρ_1); a region (2) of shock heated
driven gas, bounded by the primary shock wave (S), moving at
velocity w_s, on the "downstream" or positive x side and the
contact surface (c) (separating the driven gas from the driver
gas), moving at velocity v_2, on the "upstream" or negative x side;
a region (3) of expansion cooled driver gas; and a region (4)
of uncooled stationary driver gas at the original gas conditions
(p_4, T_4, ρ_4). Between regions (3) and (4), an adiabatic
expansion is bounded on the upstream side by the head of the
expansion fan moving upstream (i.e., toward negative x) at the
local speed of sound (a_4) and, on the downstream side, by the
tail of the expansion fan moving at speed (v_3-a_3) where v_3 is the
local gas velocity (= v_2). When (v_3-a_3) is positive, the motion
of the tail is downstream (i.e., toward positive x); cooling of
the driver gas persists into the shock tube region downstream
of the plane of diaphragm rupture. In the above, the symbols p,
T, and ρ are, as usual, pressure, temperature, and density; the
subscripts denote the region. Velocities cited are taken from a
laboratory fixed frame of reference.

| The contact surface (C in Figure 2, ideally plane and
stable) is inherently unstable[18] against small transverse
perturbations. The deviations from one-dimensionality which
turbulence implies in turbulent detonation waves are usually
thought to be a result from the shear induced by the curvature
of the discontinuity.[21] Turbulent contact surfaces also show
curvature.[20] In addition, nonuniformities associated with the

High Pressure | Low Pressure

Expansion

r_t

(T)

(C)

(S)

(H)

x
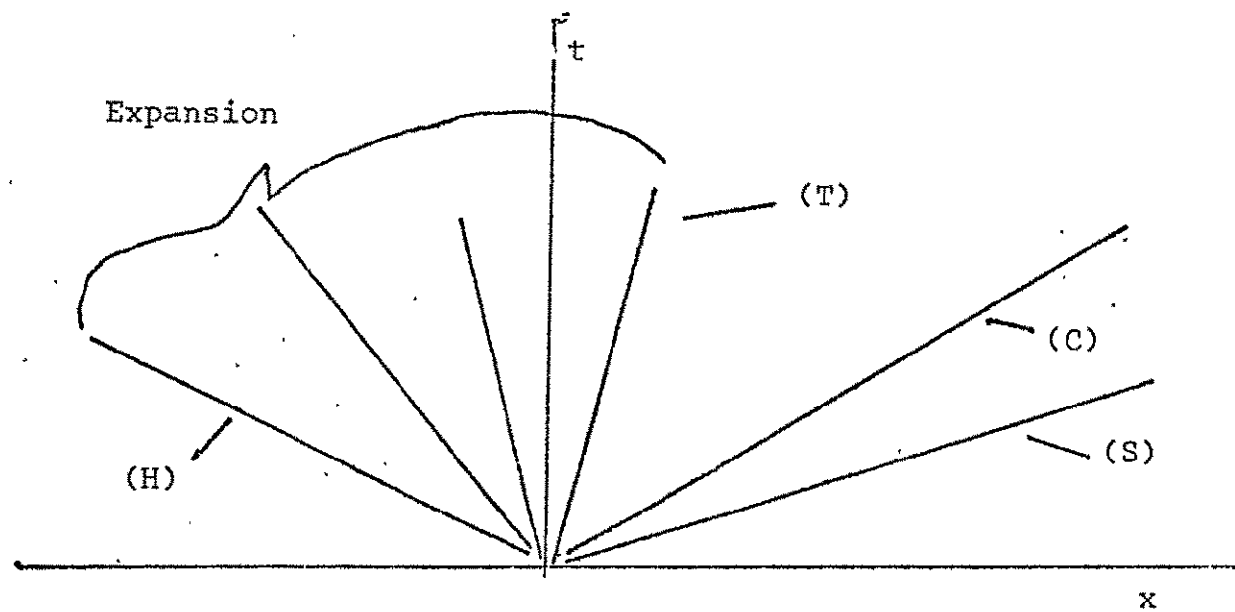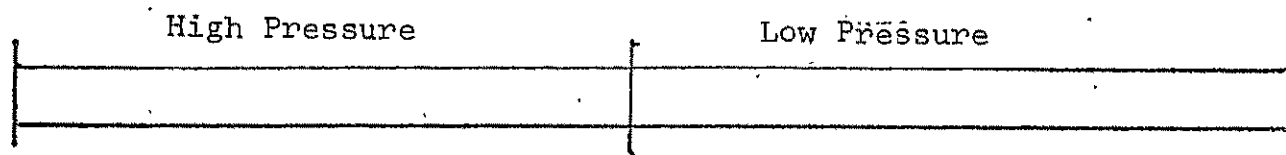
2.

rupture of a diaphragm produce perturbations in the contact
surface. Depending on the initial driver gas pressure, p_4,
the local Reynolds number at the contact surface can be made
high enough so as to be consistent with the possibility of
eventual turbulent flow. Gas particles are accelerated from
rest and acquire finally a constant downstream velocity v_3 at
the completion of the adiabatic expansion. Furthermore, in
the contact surface and wherever else the appropriate conditions
exist transition to turbulence can take place.

C. Apparatus

Our 12.7 cm diameter shock tube's test section has been placed immediately downstream of the diaphragm. High purity N_2 gas (or N_2 with various low concentration $N_2O_4 \rightleftarrows 2NO_2$ admixtures) expands adiabatically in the 153 cm long driver tube, after the rupture of a mylar partition, evolving into a low pressure region, the 336 cm long driven tube, also containing N_2 gas. The test section is 153 cm long. Pressure histories are recorded with wall mounted Kistler 606A gauges at the driver section's end plate and at five positions in the test section, including one positioned at its center.

In addition, two 2.5 cm windows at the center of the test section 76.2 cm (downstream) from the diaphragm allow an optical path through the shock tube. In some shock tube firings, this path has allowed us to use blue light absorption (0.435 μm) from a 200 Watt mercury arc lamp as a diagnostic for NO_2 density. In other firings, the optical path has been used as one. arm of a Michelson-Morley interferometer. In both cases, the basic set-up is conventional (see, for example, Bradley[22] for a discussion of these techniques). However, in the latter instance, simultaneous gas density measurements are made possible along two lines equidistant from the diaphragm and a distance one cm apart transverse to the flow of gas in the shock tube. The amplifications and the d-c supply voltages for the two phototubes which are used as fringe shift sensors are adjusted so that, for laminar unsteady low velocity flow, their signals are correlated and cancel when combined. Figure 3 shows a

F1

F2

F1

F

F2

1.9
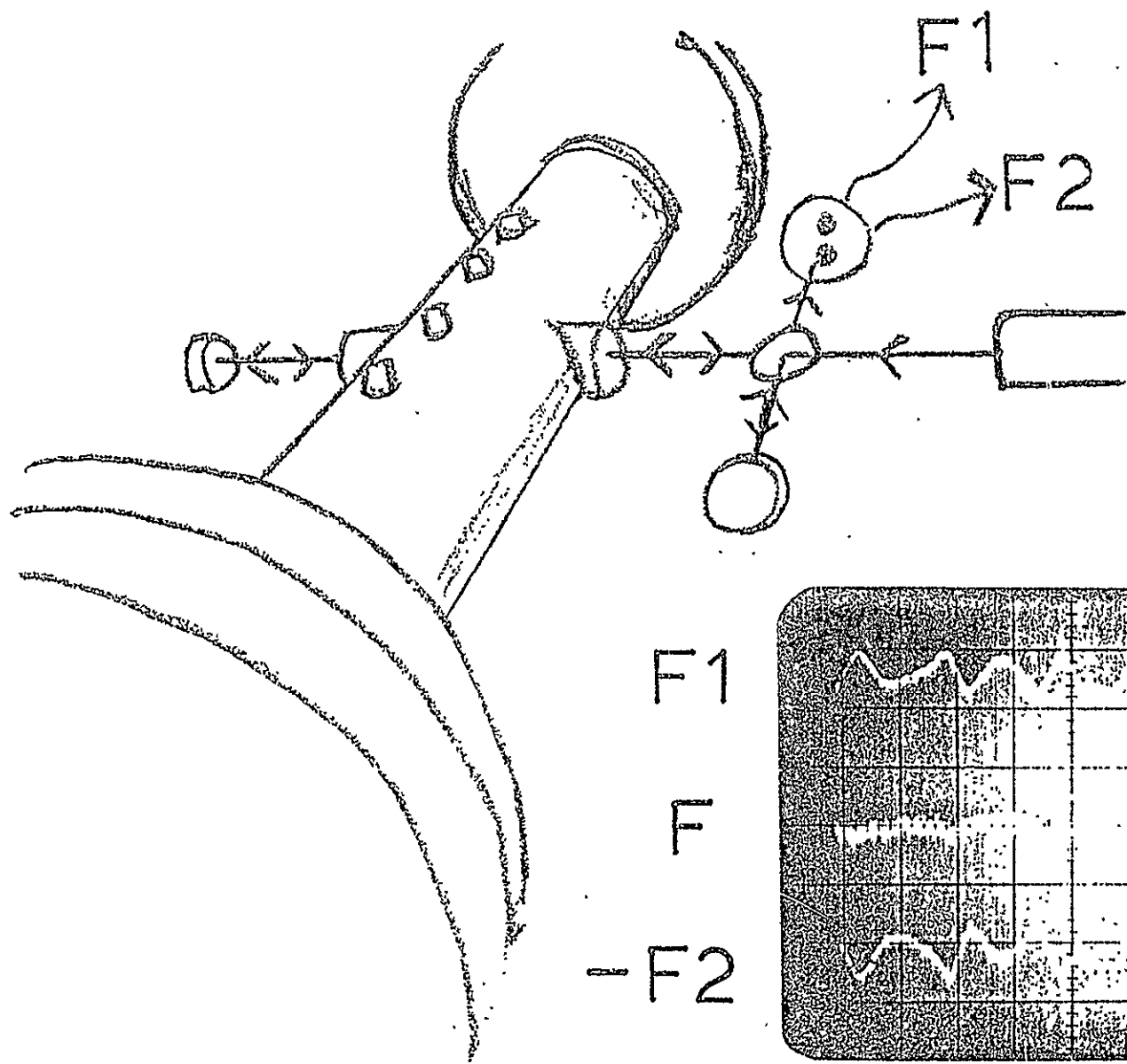
sketch of the test section and a sample of the interferometer set-up data where ordinary room air flow (resulting from the air-conditioning in our laboratory) has been used.

The pressure gauges provide start and stop signals for three time interval counters. Pressure histories, NO_2 absorption histories and the output from the interferometer (signals F1, F2 and F = F1-F2) are recorded on oscilloscopes with bandwidth > 30 MHz. A Biomation 8100 A to D converter is used in selected instances so that some of our data can be subjected to computer analyses; either the pressure history at the test section window is digitized along with the simultaneous optical data or pressure histories at two different locations are digitized.

D. Results

(i) Calibration

We have tested the use of the equations of one-dimensional gasdynamics in our experiments by comparing the measured values of the shock strength (p_2/p_1) with the values of p_2/p_1 calculated using the 1-D equations and the measured primary shock wave's Mach number, M_s $(= w_s/a_1)$. The range $1.6 < M_s < 2.1$ is chosen; for a N_2/N_2 system (N_2 as the driver and driven gas) in our shock tube, this range has given us the opportunity of varying the Reynolds numbers in the driver gas at fixed M_s since p_4 can be varied independently of p_4/p_1 (which determines M_s). Figure 4 shows the results from these comparisons. On the average, the measured values of p_2/p_1 are higher than calculated by an amount $10 \pm 11\%$; this effect gives an indication of the limit to which we might reasonably employ the 1-D equations in order to predict flow conditions in the shock wave processed driven gas.

The usefulness of the interferometer data as an indicator of contact surface arrival has also been tested. We monitor the output of one of the two phototubes on which a single fringe has been placed. We observe the sudden change in the fringe shifts to a condition of large (and apparently highly erratic) variations. We define this event as the arrival of the contact surface and measure the time interval which has passed since the arrival of the shock wave at the same point; this time interval is labeled $T_{c,m}$. For the same shock tube firing, we use the measured Mach number M_s to calculate the time interval which would be expected according to the equations of one-dimensional

$$P = (P_2/P_1)_{Meas.}$$

4.
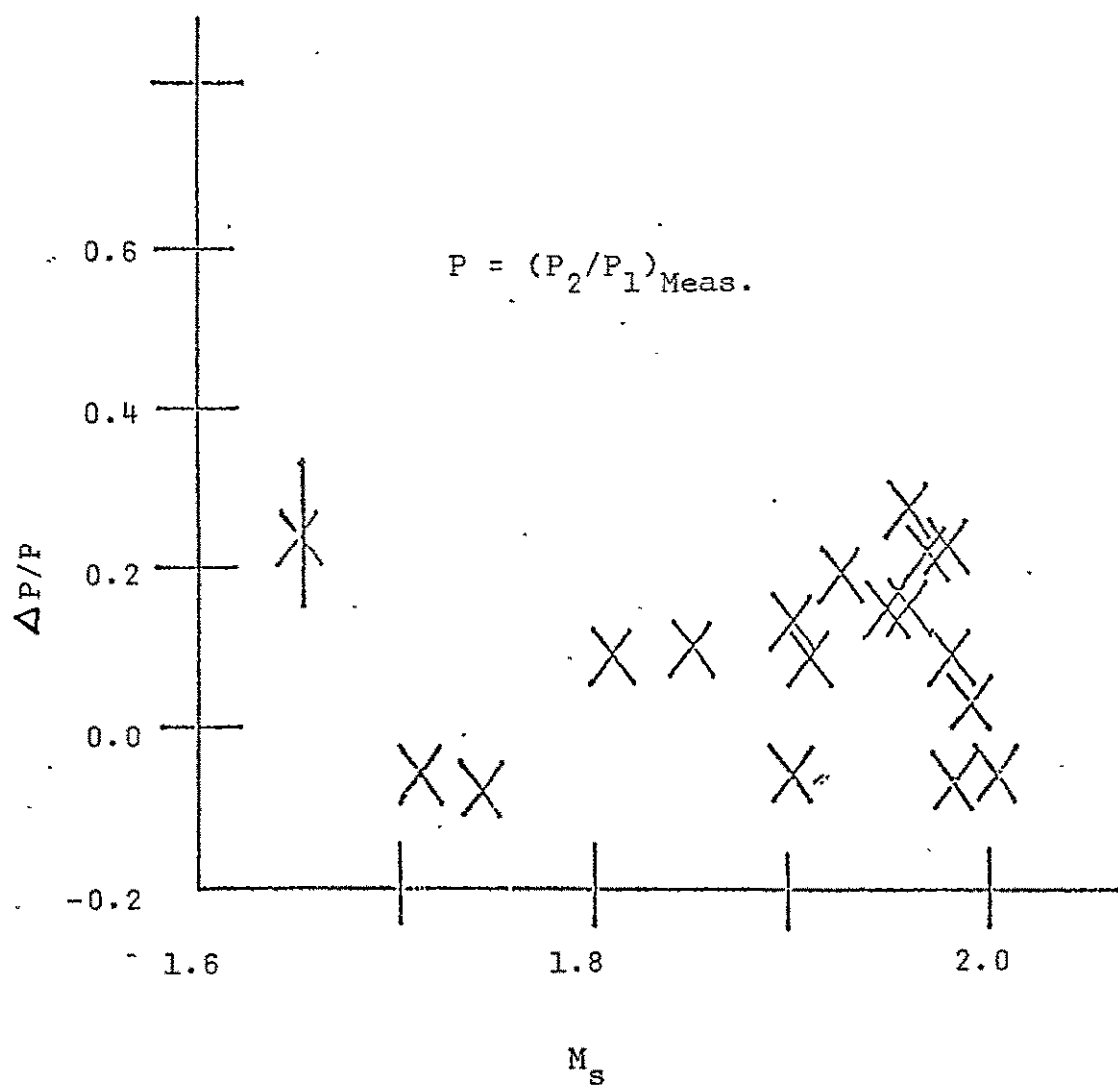
gasdynamics; this time interval is labeled $T'_{c,c}$. Figure 5 shows comparisons of $T_{c,c}$ and $T_{c,m}$. These data fit a straight line through the origin (-1 ± 42 μsec) with a slope of one (0.99 ± .08) and a correlation coefficient (r = 0.97) which is very close to one. Thus, the measured Mach number seems to be an even better predictor of the velocity of the contact surface than it is of the pressure ratio across the shock front.

In addition, we have tested the accuracy of blue light absorption as a diagnostic for NO_2 density in our set-up. For these calibrations, absorption was measured with a static fill; there was no diaphragm between the driver and driven sections of the shock tube. Lambert-Beer's Law was verified in the form:

$$\log (I/I_o) = a + b [NO_2]$$

where b = εd, ε is the extinction coefficient, d is the path length (= 13.97 cm) and $[NO_2]$ is the NO_2 density in mole/liter. We found a = -(2.9 ± 11) x 10^{-3} and ε = 149. ± 8 ℓ/mole-cm. (See Figure 6.) This value of ε is consistent with previous measurements as reviewed by Zimet.[23]

6.
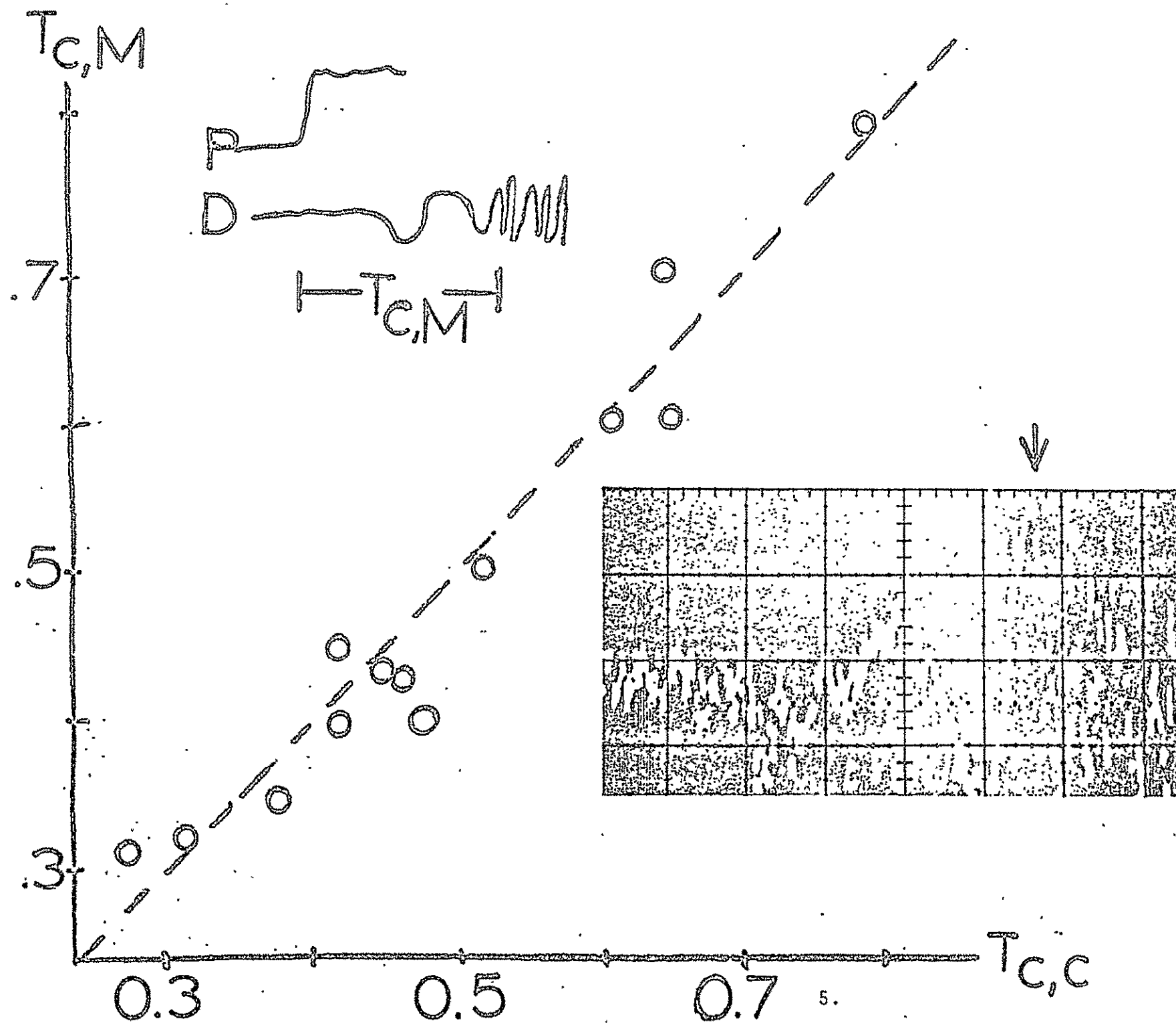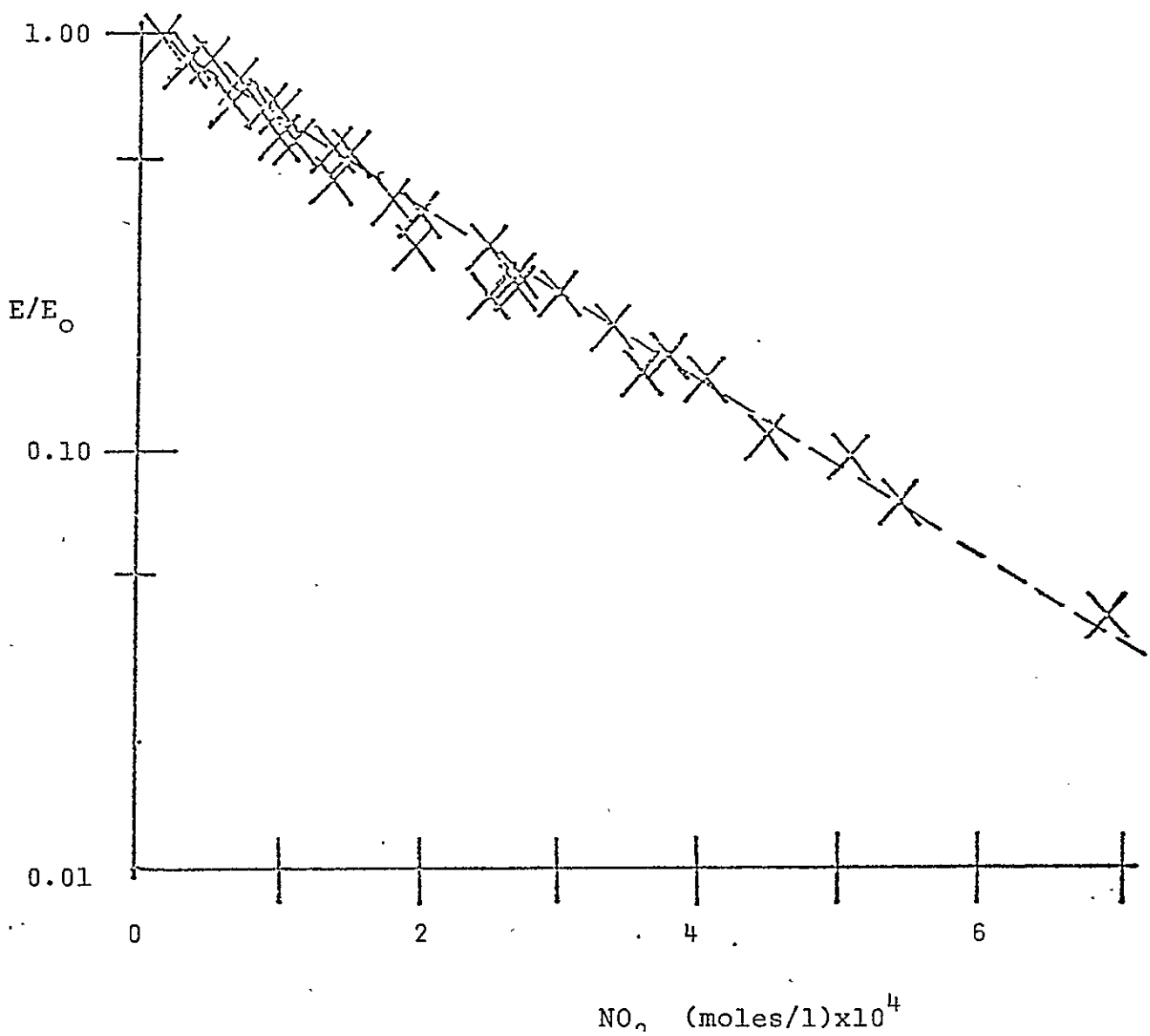
(ii)   Fluctuations

Significant localized deviations from the mean behavior are found in data from the pressure measurements, the interferometer and the blue light absorption.   These deviations are found in the driver gas, upstream from the contact surface. The pressure data show large localized variations from the average local pressure.   These are associated with a simultaneous change in the behavior of the interferometer output (F) from random variations to a behavior which shows that the density changes sensed by F1 and F2 are strongly correlated.   When a low NO_2 concentration mixture is used ($N_2+N_2O_4 \rightleftharpoons N_2+2NO_2$) as the driver gas, the pressure deviations are also associated with simultaneous large decreases in the blue light absorption corresponding to decreases in the NO_2 density.   Examples of each of these are shown in Figures 7, 8 and 9.

The statistical behavior of the pressure history has been studied.   We use the correlation function $R(\tau)$:

$$R(\tau) = \overline{p'(t)p'(t+\tau)}/\overline{p'(t)}^2 \quad ,$$

where $p'(t)$ is the difference between the instantaneous pressure and the average pressure at time t.   When we compare pre- and post-fluctuation pressure data with data obtained during the fluctuation, the correlation function appears to be structured for data in the fluctuation and unstructured for data before and after the fluctuation.   A sample of this is shown in Figure 10.   The behavior of $R(\tau)$ during the fluctuation is consistent with the behavior expected in turbulent flow.

24

(a)

(b)

(c)

(d)

P

F

t(arb. units)

8.

P

E

t(arb. units)

9.

10.
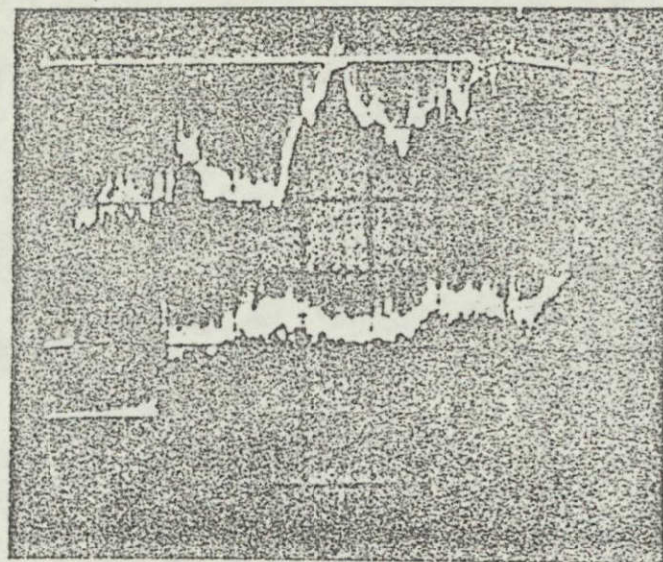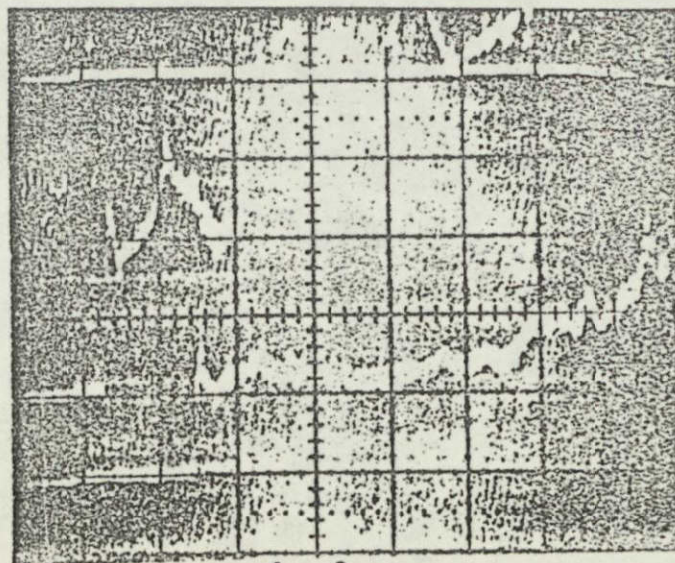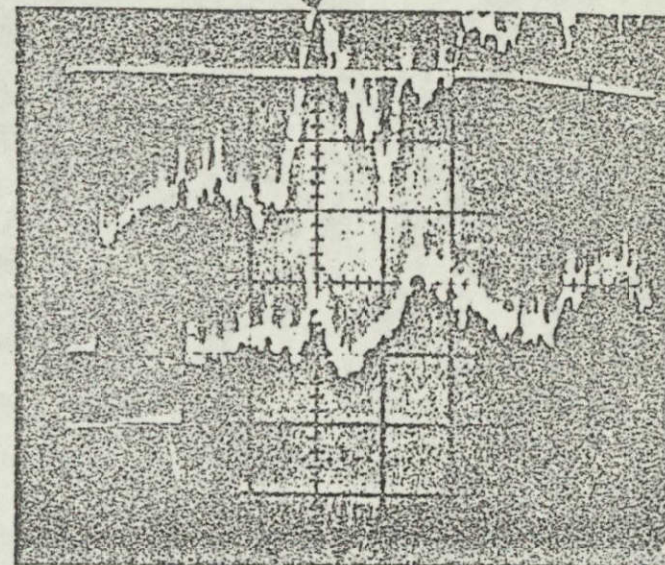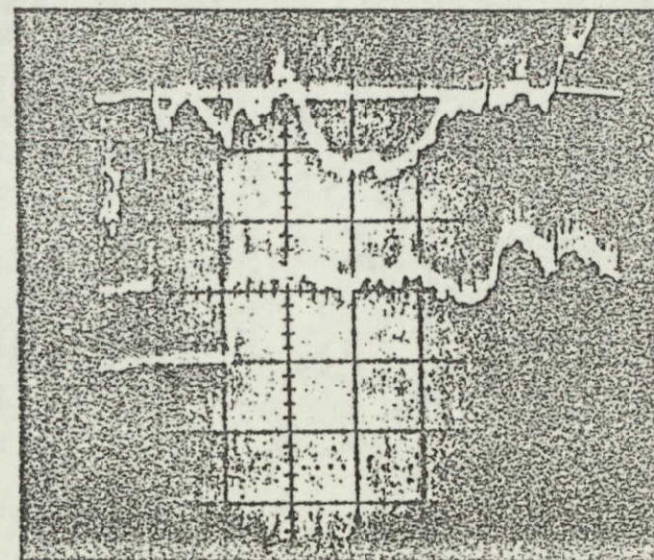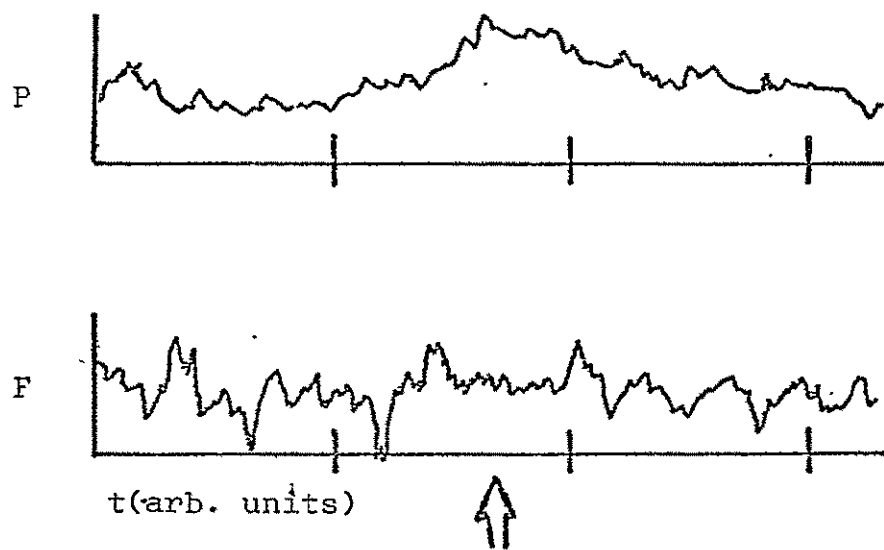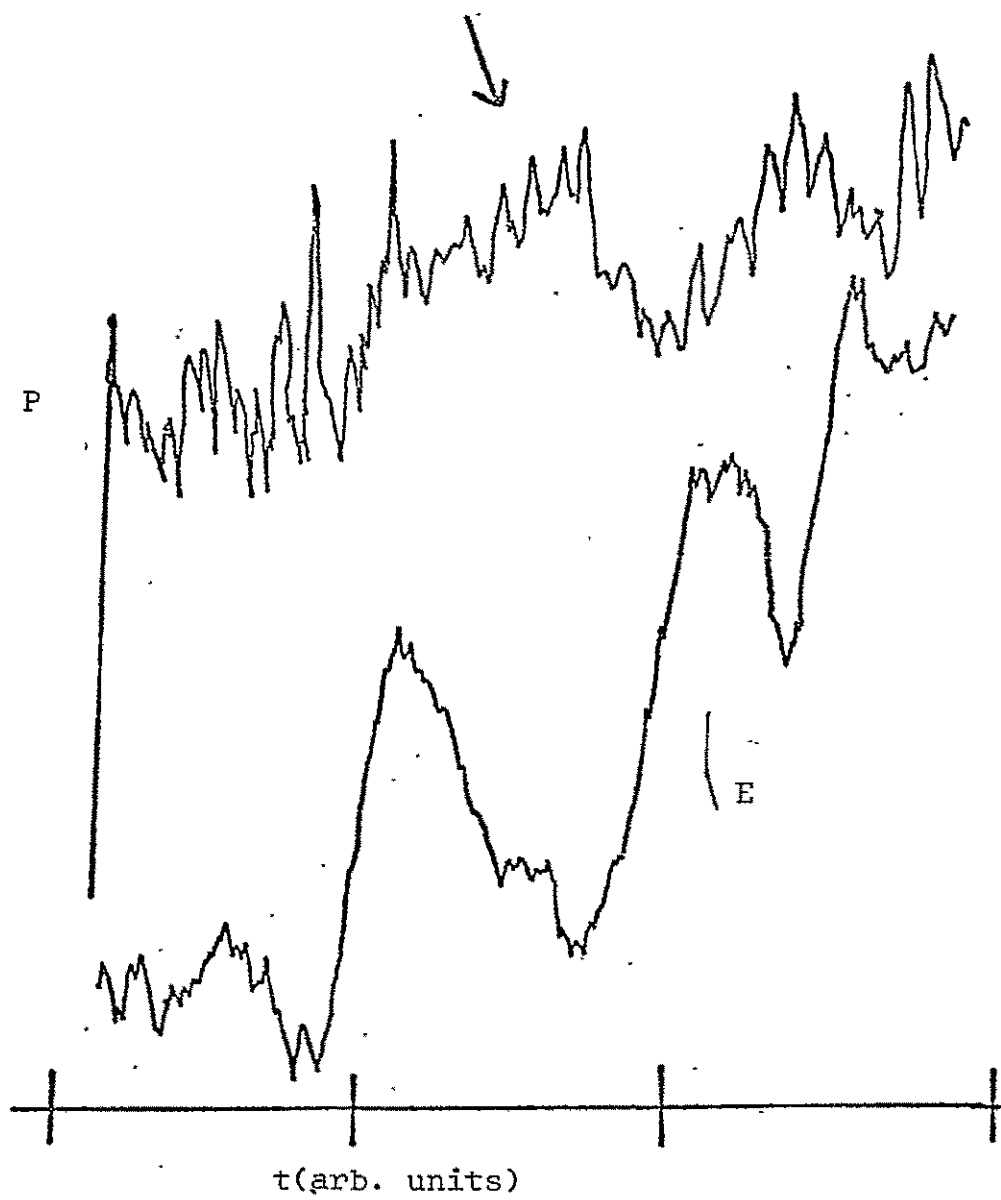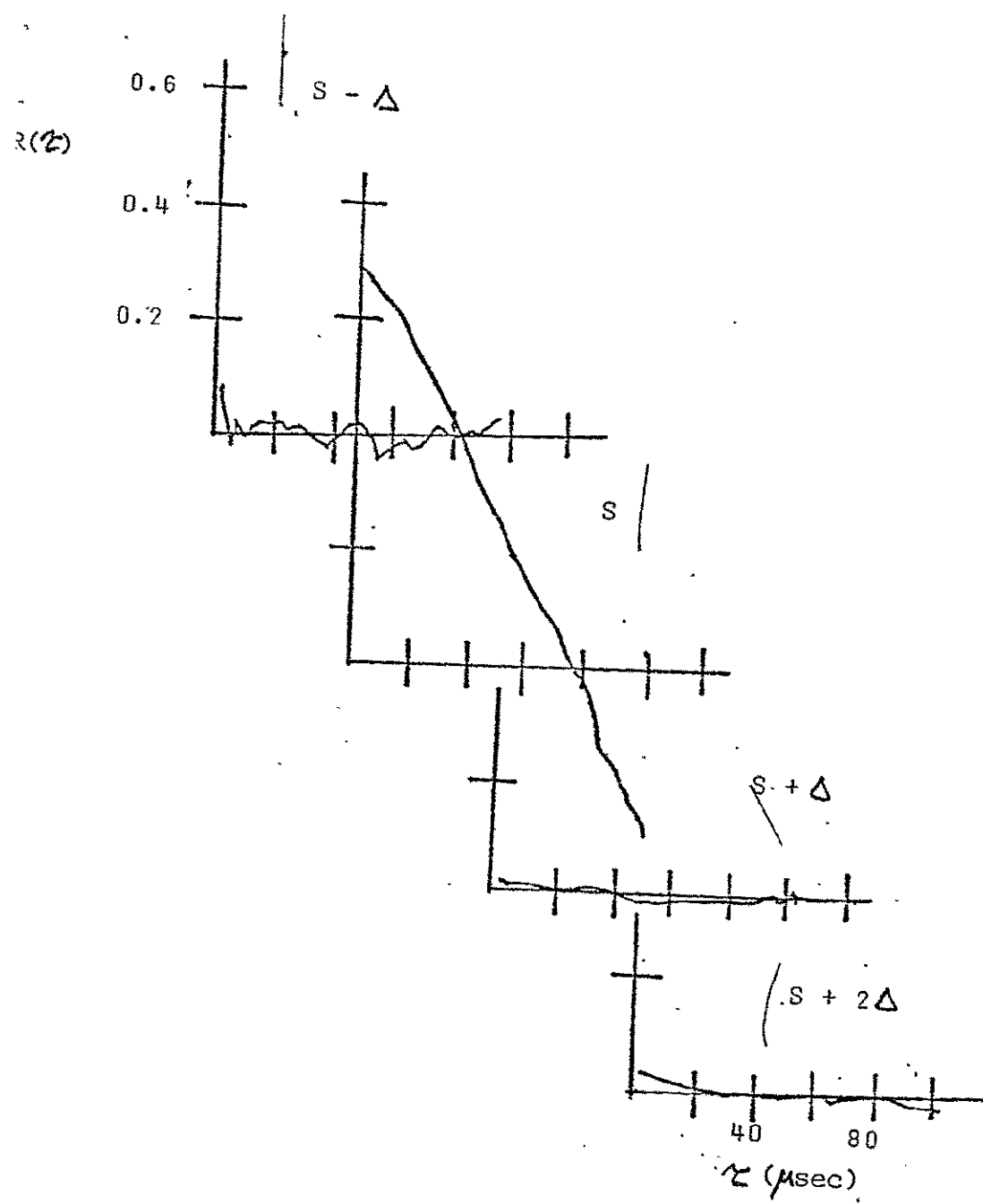
Other dynamical features of the fluctuations have been measured. At a fixed observation station, the width of the deviation in the pressure history twp (that is, its duration in the laboratory frame of reference as observed on the oscilloscope trace) appears to be relatively insensitive to increasing time lag tD behind the contact surface. This lack of trend is echoed in the duration of the signature for the fluctuation in the interferometer data twi. Figure 11a shows this for shock tube firings where simultaneous pressure and interferometer data have been obtained. Figure 11b shows that fluctuations become increasingly less noticeable with time; the magnitudes of the ratio of the peak pressure in the deviation, p_B, to the local mean pressure p_L, p_B/p_L, diminish with tD^{-1}. The speeds of several fluctuations have been measured by tracking them through consecutive pressure stations. When the measured velocities are compared with the local velocity v_3 calculated using the measure primary shock wave's Mach number M_s, we find that the fluctuations are moving at approximately 90% of the velocity of the local flow ($\Delta V/V_3 = 0.13 \pm .12$). These data are given in Figure 12. Furthermore, we have displayed in Figure 13 a Reynolds number dependence in the time lag behind the contact surface tD at which the fluctuations are detected. Shown are those shock tube firings in which two distinguishable fluctuations are discernible in the pressure history. Henceforth, the first fluctuation will be called the β_1 fluctuation and the second called β_2. The Reynolds number is based on the local flow conditions calculated for the fully expanded driver gas (region 3) in units of inverse centimeters according to

(a)

(b)

11.

12.

13.
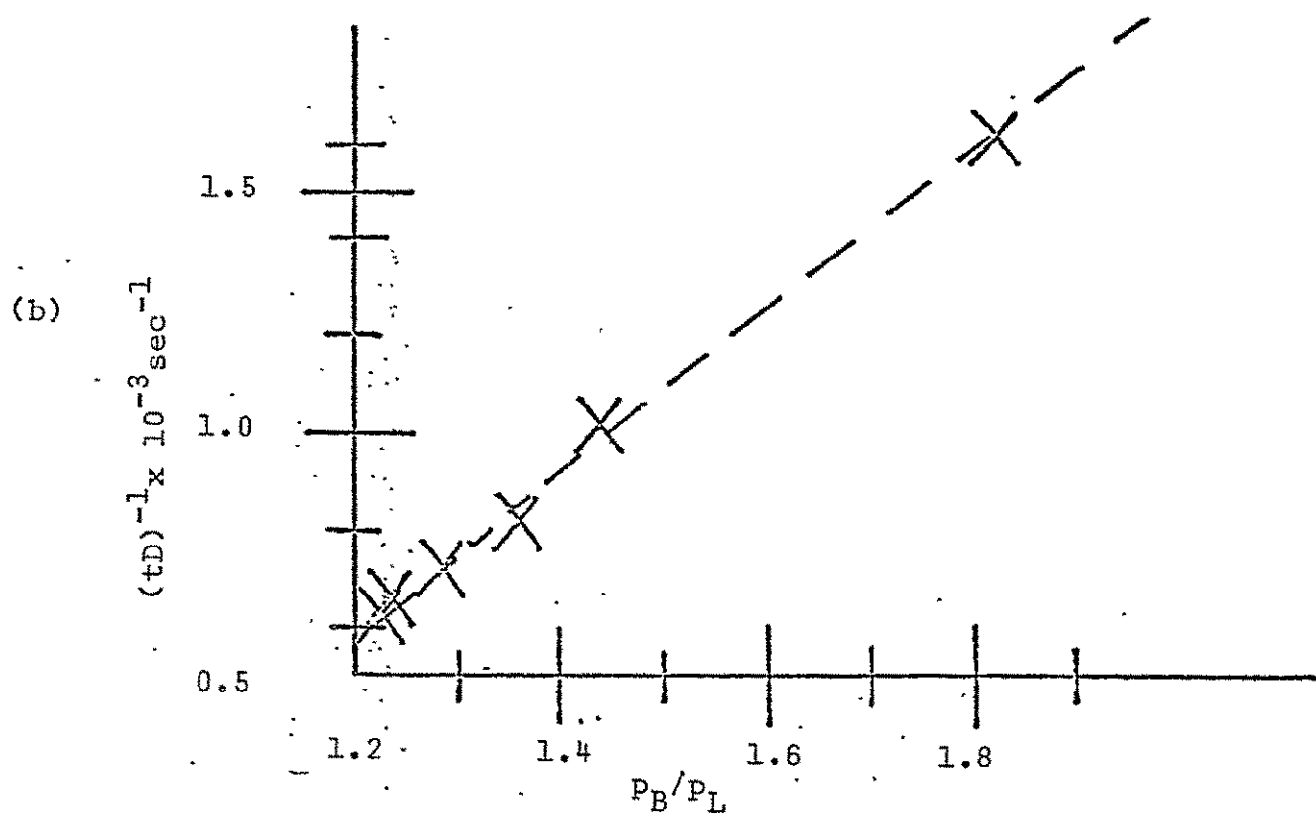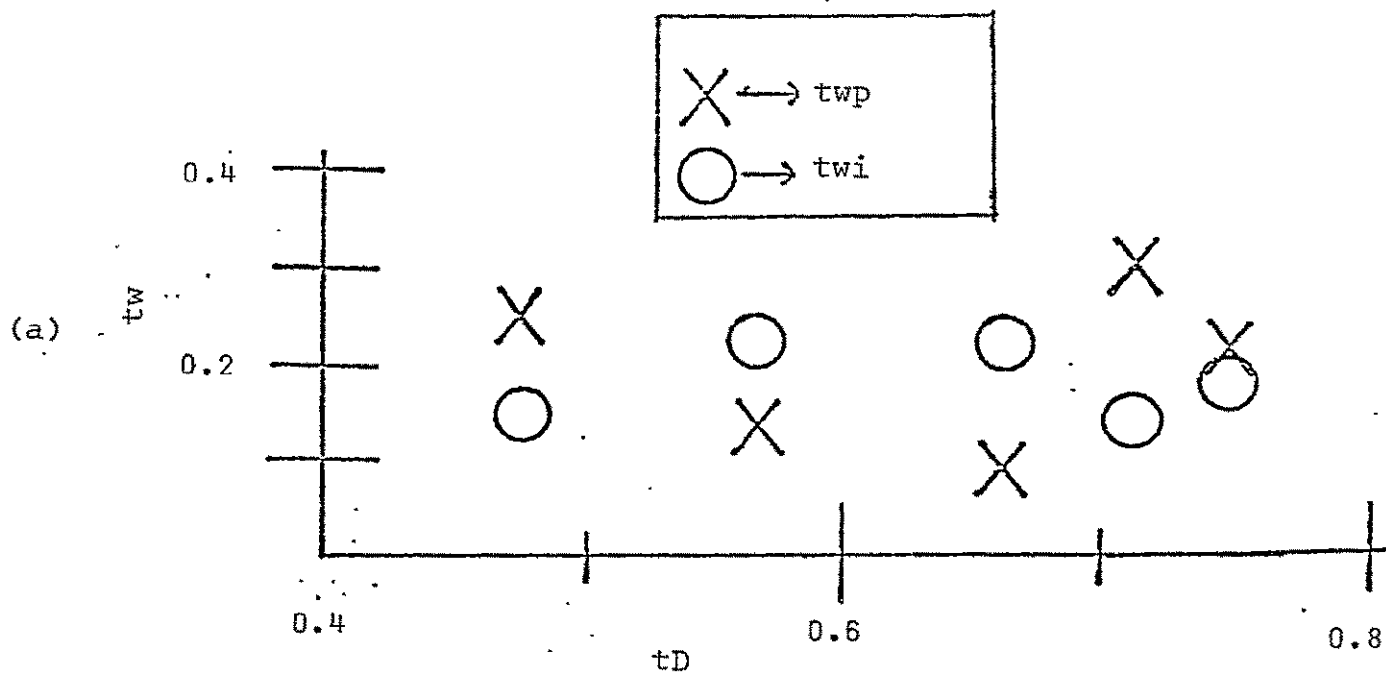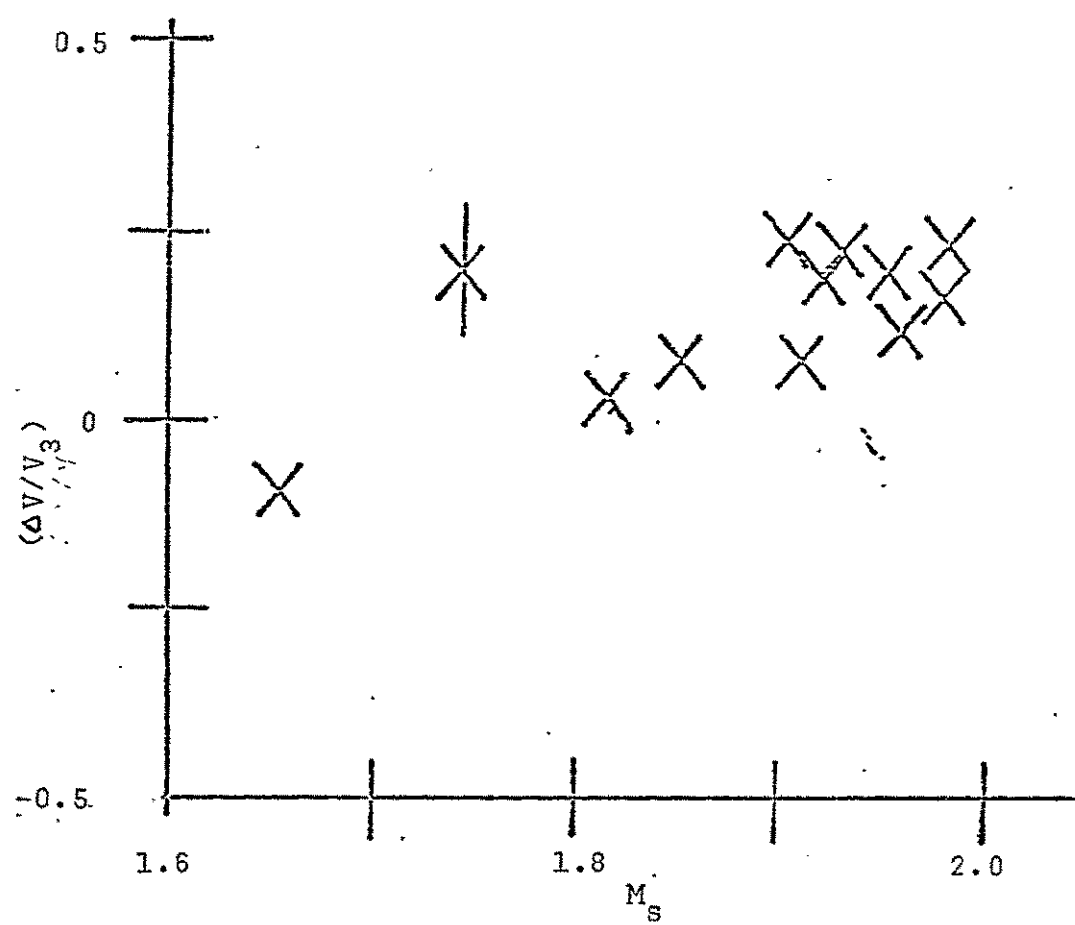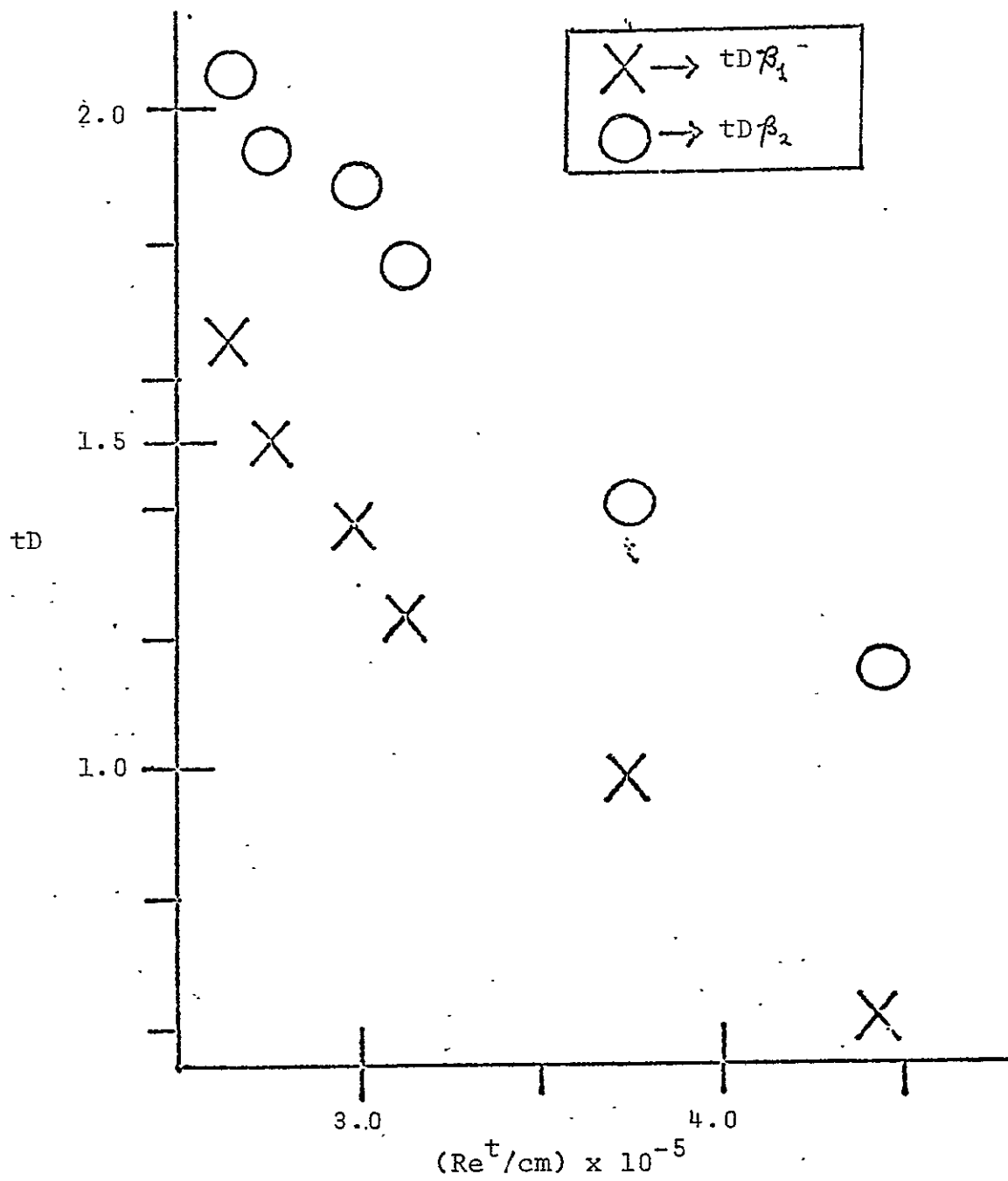
$Re^t/cm = u_3/\nu_3$ where ν is the kinematic viscosity. Notice that the separation between the fluctuations seems to be constant for all cases, 470 ± 60 μsec. Taken separately, the time lag tD decreases with increasing Re^t/cm for both the first (β_1) and second (β_2) fluctuations.

(iii) Boundary Layer Flow Model

Let's model the turbulent contact surface as a boundary layer which is transverse to and symmetric about the axis of propagation. That is, its smallest dimension is at the center of the tube; its largest dimensions (small with respect to the radius of curvature of the front) are at the shock tube's walls. This apparent boundary layer is perpendicular to the "normal" boundary layer at the shock tube's walls. The local Reynolds number in it for the observed fluctuations can be defined using: a local characteristic length,

$$\ell^* \equiv a_3 t_\tau$$

where t_τ is a characteristic turbulence time; a characteristic "free stream" transverse boundary layer velocity,

$$U_d = U_3 \ (\theta^2/8)$$

where ν is the total angle subtended by the curved contact surface; and the values of ν_3 and U_3 computed from the 1-D gasdynamical equations. From Figure 11b, we also assume that the turbulent intensity ($\equiv Q$) is related to the time lag of the fluctuations as observed at our test section according to

$$Q \ \alpha \ P_B/P_L \quad \text{and}$$

$$Q = f_i (i, \dots)(tD\beta_i)^{-1} \qquad \qquad IV-(1)$$

In this, i is a fluctuation ordering parameter and f is some (unknown) function of i whose overall value is constant in

our data once the parameter i is fixed. This assumption does
not eliminate the possibility of a M_s or U_3 dependence; in the
data for which the above relationship is confirmed (i.e., the
data in Figure 11b), the variation in M_s was only ±10%. For
consistency, our data require $f_1 < f_2$ since we find, when
$tD\beta_1 = tD\beta_2$, $(P_B/P_L)_{\beta_1} < (P_B/P_L)_{\beta_2}$.

In our case, t_τ is determined from the correlation functions
in our data, as illustrated in Figure 9, to be $t_\tau \sim 40$ μsec;
θ is estimated from the data reported in Hall[19] and Thompson[24]
to be $\theta = 0.07 \pm 0.02$ rad. The data displayed in Figure 13
(tD vs Re/cm) is replotted in Figure 14 as log $(\frac{1}{tD})$ vs Re where

$$Re = \frac{U_3\theta^2 a_3 t_\tau}{8\nu_3}$$

In both cases, a good fit to a straight line is obtained. For
the fit log $(1/tD(\beta_i)) = b_i + m_i Re_i$, we found $b_1 = -3.8$, $b_2 = -3.7$,
$m_1 = 3.3 \times 10^{-3}$, and $m_2 = 2.1 \times 10^{-3}$. Although the correlation
coefficient r is nearly one in both cases, the large uncertainty
in θ quoted above leaves the fit parameters very uncertain
($\sim \pm 50\%$). Nevertheless, Figure 14 suggests quite clearly a
linear dependence of the logarithm of turbulent intensity on
local Reynolds.

For transitional boundary layer flow, Tsugé's (1974)
solutions dictate the familiar form

$$I \equiv Q/Q_o = \exp [\phi(Re, \lambda)] \qquad\qquad IV-(2)$$

The expansion parameter λ, determined by boundary conditions,
has the units of frequency; Q_o is a minimum intensity level which
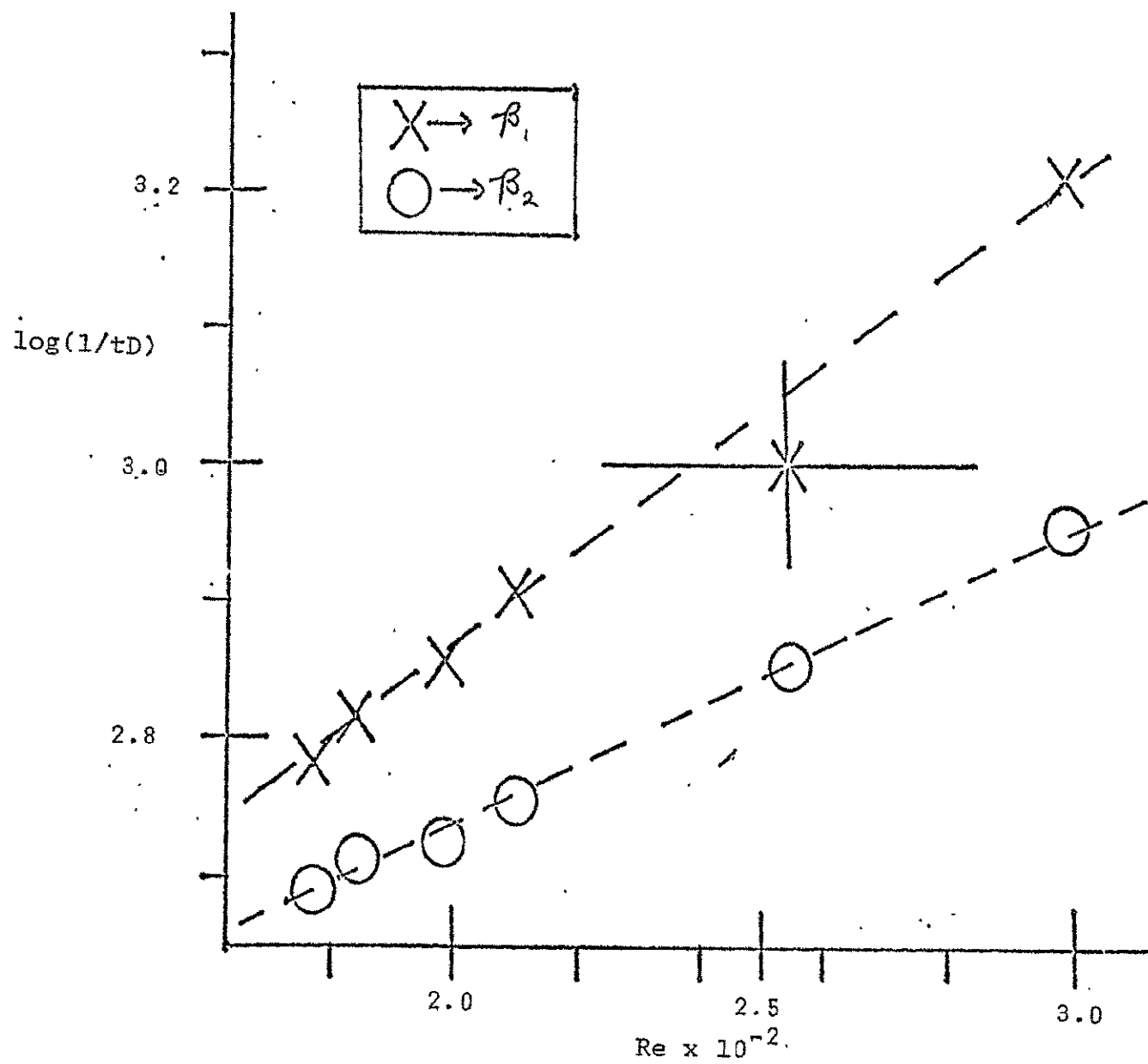
14.

must be exceeded in order for fluctuations to grow. The function ϕ has no arbitrary constants. The family of curves of I' vs Re (where I' = log I) at selected values of the nondimensional ratio $\lambda\nu/U_d^2$ shows: a first regime with $(dI'/dRe) > 0$, $(d^2I'/dRe^2) > 0$; a middle regime which is essentially linear, with $(dI'/dRe) > 0$, $(d^2/I'/dRe^2) = 0$; and a third regime with $(dI'/dRe) > 0$ initially and $(d^2I'/dRe^2) < 0$. The linear middle regime has a slope which we shall name m and an intercept on the Re axis which we name Re_n. Extrapolated to I' = 0, the middle regime gives at Re = Re_n, Q = Q_0. The third regime has a maximum in I' of value I'_{mx} at a Reynolds number which we now name Re_p. For the Reynolds number range in Figure 13, $m = 3 \times 10^{-3}$ and $(Re_p - Re_n) = (0.56) Re_p - 0.29$. Since, in the linear regime, the experimental data should fit log Q = b + (m)(Re), the determination of Re_n gives an estimate of Q_0 from:

$$\log Q \big|_{Re=Re_n} = \log Q_0 = b + (m)(Re_n) \qquad \text{IV-(3)}$$

Returning to Figure 14, and testing the approach above, one notices that both of the slopes are consistent with the theoretical value within the uncertainty of our estimates. There is no evidence of a peak in the experimental Q vs Re profile. However, since Figure 14 tells us that $Re_p > 310$ if we assume that the theory is applicable, then $Re_n > 137$ from the discussion above. Combining this inference with equations IV-(1) and IV-(3) and the fit parameters for the β_1 fluctuations we find

$$(Q/Q_0) > 4.$$

E. Conclusion

Our shock tube configuration has provided us access to
gasdynamical phenomena not previously reported. From three
different diagnostics, viz., pressure measurements, total
gas density detection, and reactant density detection, we
find localized fluctuations in the gasdynamic variables in
the driver gas flow which are distinguishable from the average
behaviors. These fluctuations show systematic changes with
Reynolds number as well as unique and reproducible features
in the nature of their characterizing parameters, viz., time
lag with respect to the contact surface, duration, deviation
strength, and statistical correlation time. These trends do
not allow the interpretation of our fluctuations as a spurious
wall boundary layer phenomena. Neither do these trends seem
affected by the variety of diaphragm materials and diaphragm
rupturing procedures used. However, the trends are not
inconsistent with an interpretation of the observed fluctuations
as examples of turbulent bursts, originating in an unstable
contact surface. In addition, attempting to interpret the
contact surface as a boundary layer in accordance with Tsugé's
kinetic theory of turbulence, we find a qualitative success
in terms of the Reynolds number trend with turbulent intensity.
We can compare the form taken by our data with the form taken
by theoretical solutions in the same Reynolds number range and
estimate that the ratio of required maximum to minimum turbulent
fluctuation level for amplifiable instabilities must be at

least four. This overall approach should therefore be extended
to and tested on a wider class of unstable gasdynamic discontin-
uities.

V. Improving the Mach Number Capabilities of Arc Driven Shock

Tubes.

A. Introduction

The pressure loaded arc driven shock tube[25] has proven to

be an invaluable tool in the study of high speed, high

temperature fluid phenomena and in the creation of high

temperature plasmas. The continuing need for improved overall

effectiveness for such devices has motivated a lot of work on

the effects of circuit inductance, capacitance, impedance

matching and driver tube configuration on the efficiency of

conversion from electrical energy to test gas kinetic energy.[25-29]

The upper limit of Mach numbers attainable from shock tubes has

correspondingly increased. However, insofar as systematic

trends in performance estimators are concerned, these previous

results have seemed to be either tentative or inconclusive.

We, therefore, have analyzed the scaling of shock tube

performance with capacitor energy, initial driver gas pressure

and driver volume. We have used data published in the literature

from ambitious and large configurations as well as the measure-

ments from our own modest facility. Specifically, we have

chosen to try to learn as much as possible from the simplest

model of the shock wave pressurizing process. With this kind of

device, we expect to ultimately produce the well-known collisional

turbulent shock fronts arising from electron recombination processes.

Section VI gives background for our planned extensions of current

diagnostic techniques for application to the shock tube environment.

B.   The Pressurization Process

In general, pressure loaded arc driven shock tubes can be thought of as operating in the same way as ordinary pressure driven shock tubes. The electric arc resulting from a spark discharge between two electrodes heats up the driver gas, uniformly increasing its pressure and temperature at constant volume. The increased pressure and driver gas speed of sound should routinely allow greater Mach numbers to be obtained in the driven tube test gas after bursting a diaphragm separating the two sections than would otherwise be obtained without preheating. We shall assume that this model adequately characterizes the pressure loaded arc driven shock tube.

The performance of the shock tube is often discussed in terms of the highest Mach numbers which it might provide. However, the speed of the shock wave obviously depends on both the driver and driven gas parameters. Therefore, one should not expect to successfully use the Mach number alone when comparing the performance of different shock tubes if there are varying configurations and operating conditions.

A more useful basis for comparison can be found as follows. Recall,[30] for a strong shock wave, that the Mach number is expressed as

$$M = \frac{\gamma_1 + 1}{2} \left(\frac{\gamma_4 T_4 \mu_1}{\gamma_1 T_1 \mu_4}\right)^{1/2} \frac{u}{c_4} \qquad \text{V-(1)}$$

where u/c_4 is an implicit solution of the equation

$$\frac{u}{c_4} = \frac{2}{\gamma_4 - 1} \left[1 - \left\{\frac{\gamma_4 (\gamma_1 + 1)}{2} \frac{\rho_1}{\rho_4} \left(\frac{u}{c_4}\right)^2\right\}^{(\gamma_4 - 1)/2\gamma_4}\right] \qquad \text{V-(2)}$$

Here, using conventional notation, γ is the ratio of specific heats, T is the temperature, μ the molecular weight, ρ the gas density, u is the gas flow velocity and c is the speed of sound. When this formulation is applied to our model, the subscript 1 refers to the quiescent driven gas and the subscript 4 refers to the driver gas after the discharge but before the rupture of the diaphragm.

The solution of Equation V-(2), $u/c_4 = F(\rho_4/\rho_1)$, can be obtained numerically. Figure 15 shows solutions for two sets of γ's. We note that $F(\rho_4/\rho_1)$ is not a very strong function of ρ_4/ρ_1. The more important parameter for increased Mach number is, therefore, T_4, which depends only on driver gas conditions.

It is well known that the cost of a shock tube facility is usually a very sensitive function of the cost of capacitors and related power supplies, i.e., on $E_c = (1/2)CV_c^2$, the energy available to the driver gas. Thus we have chosen, as a result of these considerations, to use the ratio of stored capacitor energy E_c to shock heated driver gas temperature T_4:

$$\eta \equiv \frac{E_c}{T_4} \text{ Joules/°K} \qquad\qquad V-(3)$$

as our measure for comparison. The η is an index of performance loss since an improvement in energy transfer to the driver gas shows up as a reduction in η.

15.
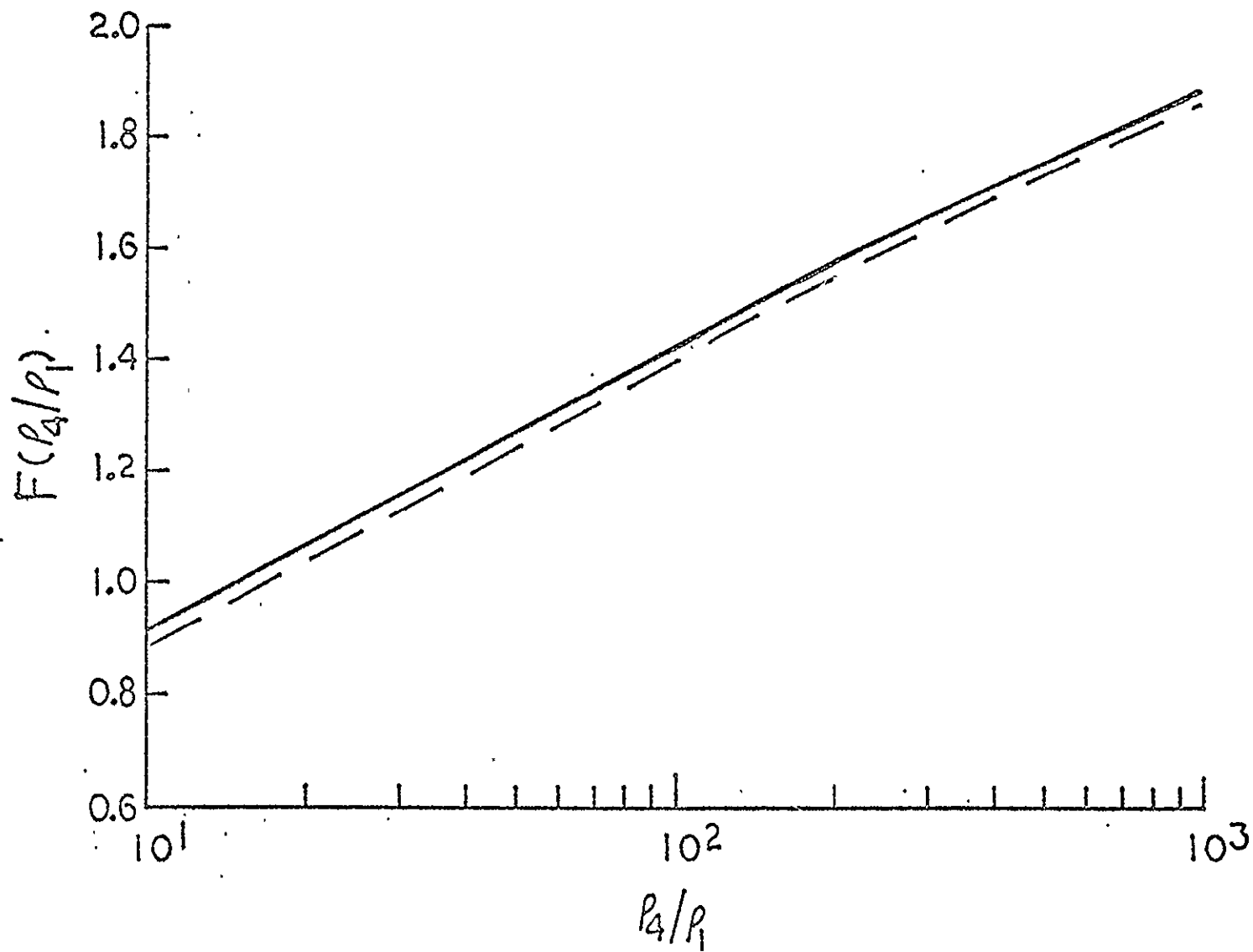
C. Performance Analyses

The most interesting configurations for these arc driven
shock tubes have been either those with cylindrical drivers
with exploding wire triggers or those with conical inserts in
the driver tubes.[29]   To us, the range of data available seemed
deficient in the low energy and small driver gas volume regime.
Our configuration has been chosen so as to begin to remedy
this deficiency.

In our set-up, we have used a cylindrical driver (with
electrodes similar to that of the conical driver), 10 cm long
with a diameter of 5 cm. Arc heating results when a 12 μF
capacitor charged to 20 kV is discharged through a spark gap
in the driver tube to which the capacitor is coupled by a
parallel plate transmission line. The luminous shock front
is monitored using piezoelectric pressure transducers, photo-
multipliers and the usual associated diagnostic electronics.
Helium and argon were used as the driver and driven gases
respectively.

(i) Dependence on E_c and V_4

The dependence of the shock speed on driver energy density
has been computed for several facilities. The results are given
in Figure 16. The speed shown there is in arbitrary units. Each
set of data has been multiplied by a constant uniquely determined
for that set; the constant was determined so that all of the
data would fall near a single line for the purpose of looking
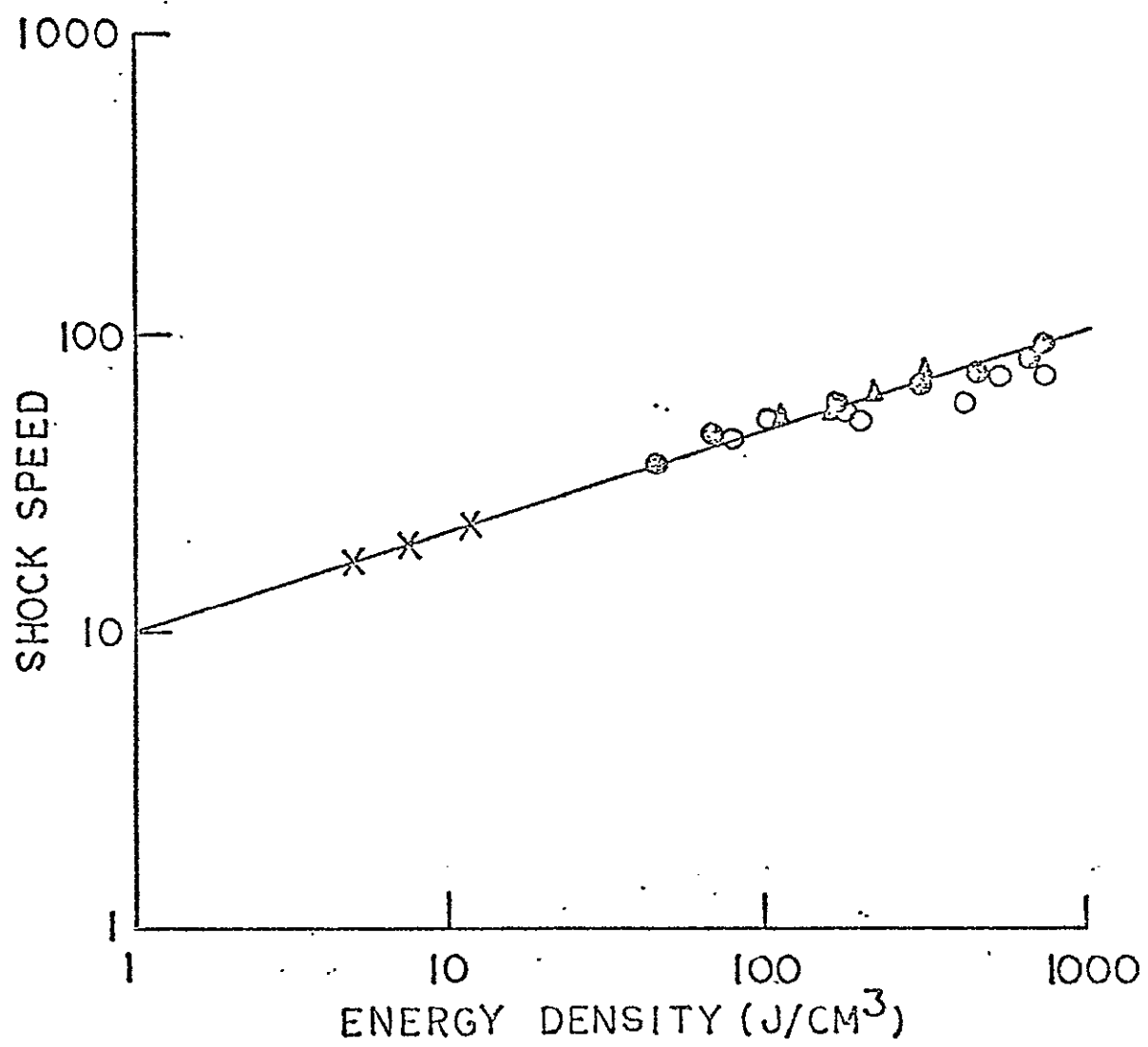
for a simple overall secular trend. It is quite evident that such a trend exists; the speed scales as $(E_c/V_4)^{1/3}$, the cube root of the ratio of stored capacitor energy to driver volume. Since $M^2 \sim T_4$ and $T_4 = E_c/\eta$, Figure 16 tells us that

$$\eta \sim E_c^{1/3} V_4^{2/3} \qquad\qquad \text{V--}(4)$$

This scaling of M with E_c may be due to increased losses at higher currents and to reduced discharge resistance at high temperature (current). The cost per unit temperature increase of driver gas increases with E_c. Notice that there is no sign of a levelling off of speed with increasing E_c even at the highest energies. Thus, it may be that the speed one obtains depends only on one's willingness to invest in bigger capacitor banks.

Figure 16 also suggests decreasing V_4 as a way of increasing performance without increasing the cost. Of course, this would be achieved at the risk of reduced time available for experimental measurements at a fixed test station location.

An unadjusted numerical comparison of some of the over-- lapping data is given in Table I. At comparable Mach numbers, one sees that increased V_4 is associated with increased η. Our driver, as would be expected, has approximately the same performance as a conical tube if a scaling of our results to the higher energies is performed.

(ii)   Dependence on Initial Driver Gas Pressure p_{4i}

For a partially ionized plasma discharge column, the resistivity will increase with increasing gas pressure. Increasing the resistance of the gap will increase the energy dissipated in the arc. A higher pressure, on the other hand, dissipated in the arc. A higher pressure, on the other hand, will mean a greater mass of driver gas, a larger driver gas heat capacity, i.e., a reduced T_4, and therefore an increased index of performance loss η.

We have measured the dependence of Mach number M on p_{4i}. The results are shown in Figure 17. Note the existence of a plateau at high pressure where M becomes effectively independent of p_{4i}. Since $T_1 = T_{4i}$ in our set up then $p_{4i} = RT_{4i} = RT_1$. The fractional efficiency of electrical energy conversion is given by

$$\varepsilon = \frac{3}{2} \rho_4 RT_4 / (E_c/V_4)$$

and we find, using equation V-(1),

$$M \sim (\frac{\varepsilon}{p_{4i}})^{1/2} F(\rho_4/\rho_1)$$

Thus, the plateau in Figure 17 implies that the resistivity does not increase with pressure fast enough to compensate for the increased heat capacity of the driver gas.

This inference is confirmed in Figure 18. The measured efficiency is explicitly shown to have a dependence on p_{4i} which is too weak for an increase in p_{4i} to be reliably impactful on the Mach number.

17.

18.
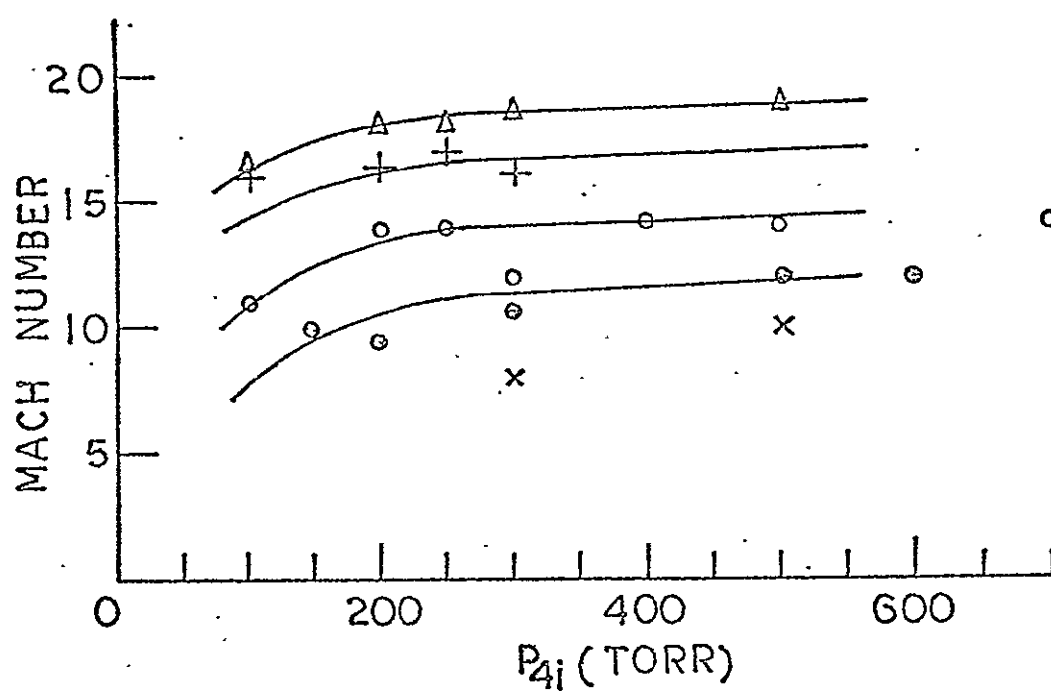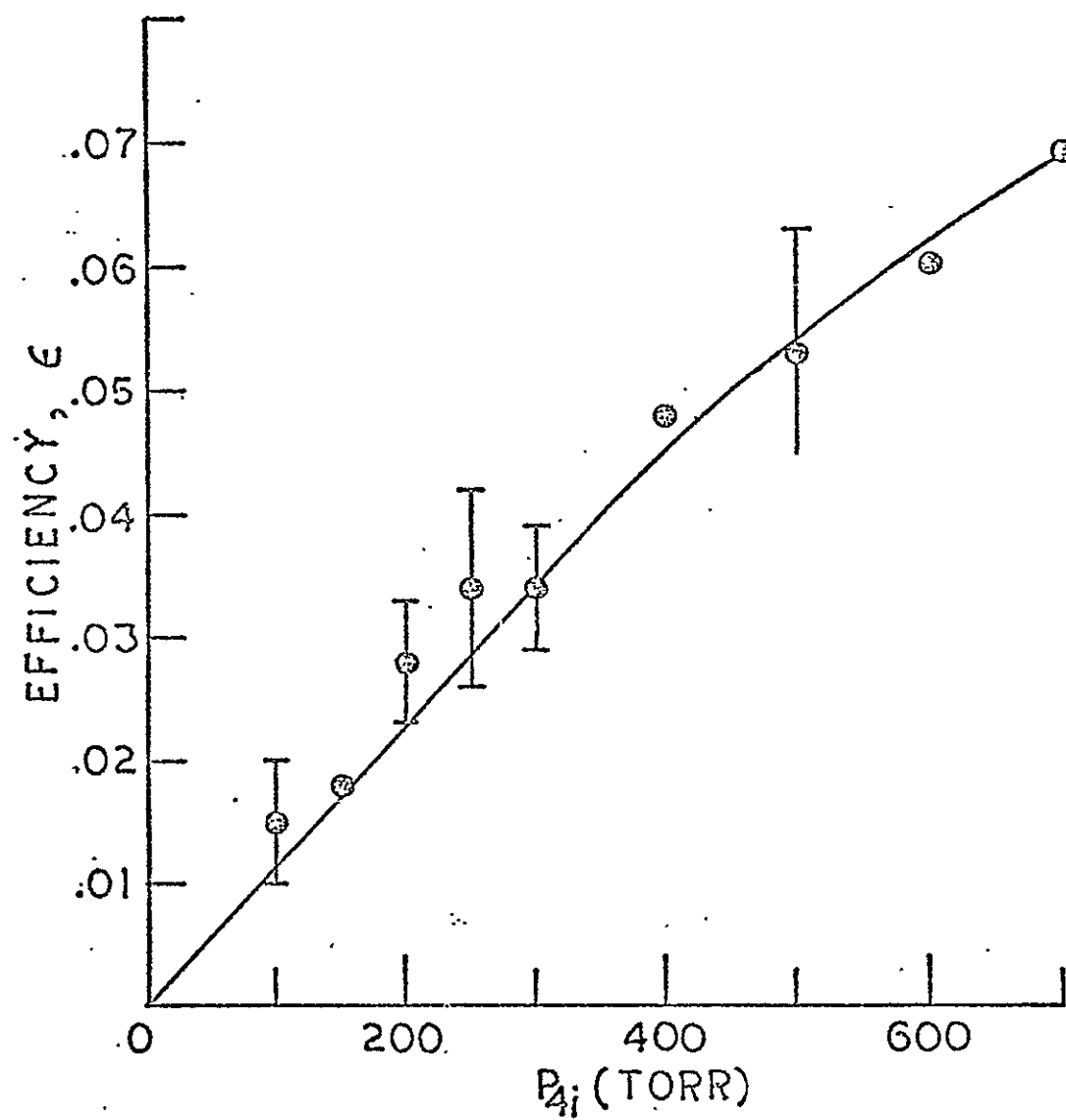
Thus, it seems unnecessary to operate the pressure loaded arc driven shock tube at the ultra high driver pressures which are now often used unless there is some motivation other than improved performance. Reduced driver pressure, wherever possible, will eliminate the need for heavy and expensive driver tube designs. The lower limit for p_{4i} should be determined, insofar as these analyses are concerned, by the need for reproducible flow properties.[31]

(iii) Dependence on Discharge Length

The total resistance of a discharge is directly proportional to its length. For maximum energy dissipation in the arc, the length of the arc should be maximized for a given driver volume. Using the efficiency as defined above,

$$T_4/T_1 = 1 + (\tfrac{2}{3} \, \varepsilon E_c/V_4)/(\rho_4 R T_1) \ .$$

At large Mach numbers,

$$T_4/T_1 = (2 \, E_c)/(3V_4 \rho_4 R T_1) = E_c/\eta T \ .$$

Hence

$$\varepsilon = (3V_4 \rho_4 R/2\eta) \sim (\frac{E_c}{V_4})^{-1/3} \qquad\qquad \text{V-(5)}$$

and we see that the efficiency of conversion increases with increasing discharge length ($\sim V_4^{1/3}$).

Nonetheless, increasing V_4 to increase resistance and conversion efficiency is not advisable since it decreases the total energy density and increases the index of performance

loss η as discussed above. Attempts, therefore, to improve performance through the management of driver gas circuit resistance inevitably encounter a fundamental handicap.

D. Conclusions

A simple physical model of shock wave pressurization has apparently succeeded in providing a consistent overall interpretation of the performance of a variety of pressure loaded arc driven shock tubes. This model should be limited only in the sense that the equations of one-dimensional gas dynamics are usually limited in shock tube analyses.[30] Within these limits: The change in electrical conversion efficiency resulting from a change in the initial driver gas pressure does not significantly improve either the performance or the highest Mach number available from a given facility once a nominal operating p_{4i} has been ascertained. Secondly, increasing the discharge volume so as to increase the conversion efficiency represents a change in configuration in this model. Table I, equation V-(4) and equation V-(5) show that such a change can be expected to have a complicated effect on the index of performance loss (and cost) and Mach number; the driver volume should be the smallest compatible with experiment test times and the discharge length should be maximized for this volume. However, at constant volume, Mach numbers and the index of performance loss can be expected to increase (while the efficiency decreases) indefinitely with the cube root of capacitor energy.

TABLE I

TYPE	E_c	p_{4i}	$V_4 (cm^3)$	M	T_4/T_1	p_1 air	η	Ref.
cylindrical driver; spiral trigger wire	1 MJ	27.2 atm He	5969	44 / 34 / 22	44 / 41 / 37	1 Torr / 10 Torr / 100 Torr	7 MJ/K	1
cylindrical driver; straight trigger wire	1 MJ	27.2 atm He	5969	30 / 23 / 16	21 / 19 / 20	1 Torr / 10 Torr / 100 Torr	15 MJ/K	1
conical driver	235 kJ	9.2 atm He	632	55 / 39	57 / 42	0.1 Torr / 1 Torr	1.4 MJ/K	2
conical driver	290 kJ	11.8 atm He	350	85 / 80	132 / 68	0.05 Torr / 0.05 Torr*	0.7 MJ/K / 1.3 MJ/K	3
Present work cylindrical driver spark gap triggered	2.4 kJ	less than 1 atm He	203	14 / 19	8 / 8	1 Torr* / 0.1 Torr*	0.09 MJ/K	-

* Driven gas is argon.

54.

VI. Resonant Absorption in an Argon Plasma at Thermal Equilibrium

A. Introduction

The absorption of resonant argon ion laser radiation has been proposed as a sensitive temperature diagnostic for an argon plasma in thermal equilibrium.[32] Absorption as a diagnostic for electron density and temperature is a well established technique in plasma physics that supplements other well known techniques such as line and continuum intensity measurements, probe measurements, light scattering, interferometry, and so on. The main advantage of absorption is in its relative simplicity; all that is needed is a light source (laser) and a detector without the need for calibration. On the other hand, absorption coefficients depend on both the density and the temperature and, their use has the disadvantage of requiring an independent measurement of one of these parameters. The calculation of the absorption coefficient in Ref. 32 has two noteworthy features: (1) the absorptions due to free-free transitions (inverse bremsstrahlung) and bound-free transitions (photo-ionization) are neglected; (2) the effects of Doppler broadening at low densities and high temperatures are not included. In this paper, we present calculations of the absorption coefficient of an equilibrium argon plasma at $\lambda = 4880 \overset{o}{A}$ without the limitations just mentioned and evaluate their implications.

B.   Theoretical Approach

(i)   The Absorption Coefficient

The total absorption coefficient K can be expressed as

$$K = (1-(\nu_p^2/\nu^2))^{-\frac{1}{2}} (K_{ei} + K_{en} + K_p + K_r) \qquad VI-(1)$$

where ν_p is the plasma frequency and ν is the laser frequency.
The contributions to K indicated in equation VI-(1) come from
the following contributions: (i) electron inverse bremsstrahlung
in the presence of an ion, I_{ei}; (ii) inverse bremsstrahlung by
electrons during collisions with neutral atoms, K_{en}; (iii) photo-
ionization of highly excited atoms and ions, K_p; (iv) resonance
absorption of 4880 Å radiation by Ar II ions, K_r. Let's now
explicitly characterize these contributions.

The absorption coefficient due to electron inverse
bremsstrahlung in the field of ions is:[33]

$$K_{ei} = \frac{3.692 \times 10^8 n_e}{\nu^3 T^{1/2}} (n_{11} + 4n_2 G_2) \qquad (c.g.s.) \quad VI-(2)$$

where n_e, n_1, and n_2 are the electron, singly charged ion and
doubly charged ion densities respectively, T is the temperature
and G_1 and G_2 are the Gaunt factors. The value of G_1 is taken
from Schnapauff's work[34] ($G_1 = 1.7$) and G_2 is assumed to be equal
to one; the Gaunt factors are relatively insensitive to the
temperature.

The electron-neutral atom inverse bremsstrahlung absorption

coefficient can be approximated by the expression used in microwave theory:[35]

$$K_{en} = \left(\frac{e^2 n_e}{\pi m_e c}\right) \nu_{eff} \left\{\nu^2 + (\frac{\nu_{eff}}{2})^2\right\}^{-1} \qquad \text{VI-(3)}$$

where $\nu_{eff} = n_o <v\sigma_{tr}>$. Here, n_o is the density of neutrals and $<v\sigma_{tr}>$ is the product of the electron velocity and the cross-section for momentum transfer collisions averaged over a Maxwellian distribution. The value of σ_{tr} used in the calculation is taken from Engelhardt and Phelps.[36] The validity of using equation VI-(3) in the visible wavelengths is discussed in numerous papers dealing with laser induced gas breakdown.[37] We notice in passing that the electron inverse bremsstrahlung absorption during collisions with neutrals is generally much smaller than the electron inverse bremsstrahlung except when the degree of ionization is less than 1%.

The contribution of photoionization of highly excited atoms is:[33]

$$K_p = \frac{2C_1 T}{\nu^3} \left\{ \frac{n_o g_1^o}{U_o} \exp\left(-\frac{(\chi_1 - \Delta\chi_1)}{KT}\right)\xi_1 + \frac{4g_2^o n_1}{U_1} \right.$$

$$\left. \times \exp\left(-\frac{(\chi_2 - \Delta\chi_2)}{KT}\right) \xi_2 \right\} \{\exp(\frac{h\nu}{KT})-1\}, \qquad \text{VI-(4)}$$

$$C_1 = \frac{16\pi^2 K e^6}{3\sqrt{3}\ C h^4} = 8.915 \times 10^{23} \quad \text{(c.g.s.)}$$

where g_1^o and g_2^o are the statistical weights of the ground states, of the first and second ionization stages respectively and U_0 and U_1 are the partition functions of Ar I and Ar II with ionization potentials χ_1 and χ_2. The lowering of the ionization potentials due to the plasma charges are $\Delta\chi_1$ and $\Delta\chi_2$. The factors are. essentially independent of temperature in the visible region of the spectrum. The values for ξ_1 obtained by Schlüter[38] and the values of ξ_2 as measured by Vorpaeva[39] are used in our calculations.

The absorption coefficient of radiation in resonance with this transition is

$$K_r = \frac{h\nu}{c} B_{\ell u} \phi(\nu) (n_\ell - \frac{g_\ell n_u}{g_u}) = \frac{h\nu}{c} B_{\ell u} \phi(\nu) n_\ell (1-\exp(-h\nu/KT)) \quad \text{VI-(5)}$$

where $B_{\ell u}$ is the Einstein B coefficient and $\phi(\nu)$ is the absorption lineshape. In this, we define n_ℓ, n_u and g , g_u as the densities and statistical weights of singly charged ions in the lower $(4s^2P_{3/2})$ and upper $(4p^2D_{5/2})$ levels of the 4880 Å argon ion transition, i.e.,

$$n_\ell = \frac{g_\ell}{U_1} n_1 \exp(-E_\ell/KT), \quad n_u = \frac{g_u}{U_1} n_1 \exp(-E_u/KT). \quad \text{VI-(6)}$$

The reduction in absorption due to stimulated emission is incorporated in equation VI-(5) and the laser linewidth is assumed to be much smaller than the absorption linewidth.

The lineshape factor $\phi(\nu)$ is a convolution of the (Gaussian or) Doppler and Stark broadened lineshapes. Doppler broadening will dominate in the low density, high temperature case while Stark

broadening will dominate at high densities. A large fraction of the total Stark broadening will be due to electron impacts which produce a Lorentzian lineshape about a central frequency, ν_o:

$$\phi_s(\nu) = \frac{(\Delta\nu_s/2\pi)}{(\nu-\nu_o)^2 + (\Delta\nu_{s/2})^2} \qquad \text{VI-(7)}$$

where $\Delta\nu_s$ is the full width at half intensity maximum.

In addition to Stark broadening, there is a Stark shift of the line center, so that ν_o and the laser frequency ν_L do not coincide, of magnitude d where

$$\nu_o = \nu_L + d \qquad \text{VI-(8)}$$

Neglecting ion broadening, $\Delta\nu_s$ and d will be proportional to n_e and are weak functions of the temperature. (The validity of the above assumptions concerning Stark effects and the errors involved therein are fully discussed in Ref. 32.) The linewidth and the Stark shift parameters for the 4880 Å transition are taken from the measurements by Labat.[40]

At very high electron densities, other Ar I and Ar II lines may be sufficiently broadened and shifted so as to contribute to the absorption. However, since this only occurs when the bremsstrahlung and photoionization dominate, resonance absorption at the far wings of other lines is not included in our calculations given below.

(ii) Plasma Composition

For a fixed electron or gas density, the plasma composition is computed using the Saha equation and the laws of conservation of mass and charge. We satisfy conditions for local thermo-dynamic equilibrium as discussed by Griem.[41] The lowering of the ionization potentials is determined from the Debye-Hückel theory:[35]

$$\Delta\chi_m = \frac{me^2}{\lambda_o} \qquad\qquad \text{VI-(9)}$$

The partition functions are computed by summing over the energy levels tabulated by Moore[42] and integrating over assumed hydro-genic levels for the excited states close to the continuum. The same ionization potential lowering is assumed in the calculation of the partition functions and in the Saha equation. Plasma composition is computed through the first, second, and third ionization stages (i.e., for n_1, n_2, and n_3).

(iii) Limitations of Theory

The limits on the laser power to be used, W, have been discussed in Ref. 32. The lower limit is determined by the ratio of the transmitted laser power to the spontaneous emission. The upper limit is determined by the condition that the absorption of laser energy should not appreciably perturb the system. The result in Ref. 32 is:

$$2 \times 10^{-21} \, n_e \leq W \leq 4 \times 10^{-32} \, n_e^2$$

for a beam diameter of 2 mm and a beam divergence of 10^{-5} sterad.

The criterion for the existence of LTE has been formulated

by Griem. To insure collisional domination up to the ground
state, the radiative population rate of the ion ground state from
the resonance level should be 10 times smaller than the corres-
ponding collisional population rate. This translates to[32]

$$n_e \gtrsim 2 \times 10^{13} T^{1/2} cm^{-3}.$$

Furthermore, calculations based on a collisional radiative model[12]
indicate that at $n_e = 10^{16}$ cm^{-3} LTE exists for the Ar I and II
systems at an electron temperature of $T_e = 4$ eV. Since the lower
the temperature the easier LTE is reached, then LTE is assured
when $n_e \gtrsim 10^{16}$ cm^{-3} and $T_e \lesssim 25,000$ K.

The assumption of Stark broadening by electron collisions
only entails an error of about 8% for $10^{16} \lesssim n_e \lesssim 10^{21}$ cm^{-3} and
$10,000 \lesssim T \lesssim 40,000$ K.[32] Finally, the assumption of a very narrow
laser line compared to the Stark broadened absorption line is
easy to satisfy with commercial argon ion lasers especially if
one uses a single mode laser.

C. Applications: 10^{16} cm^{-3} $<n$ $<10^{19}$ cm^{-3}; 10^4 °K $<T$ $<2.4(10^4)$ °K

Using equation VI-(1), the absorption coefficient K is first
computed as a function of temperature at fixed gas densities
n ($=n_0 + n_1 + n_2 + n_3$). The results are shown in Figure 19. Foe
a fixed n, we see that absorption is sensitive to temperature.
However, for $n \lesssim 10^{17}$ cm^{-3}, the absorption coefficient is not a
single valued function of the temperature. This is due to a
shift in the balance between the free-free and bound-free
transitions' contributions to the absorption coefficient (which
decrease with increasing temperature) and the contribution from
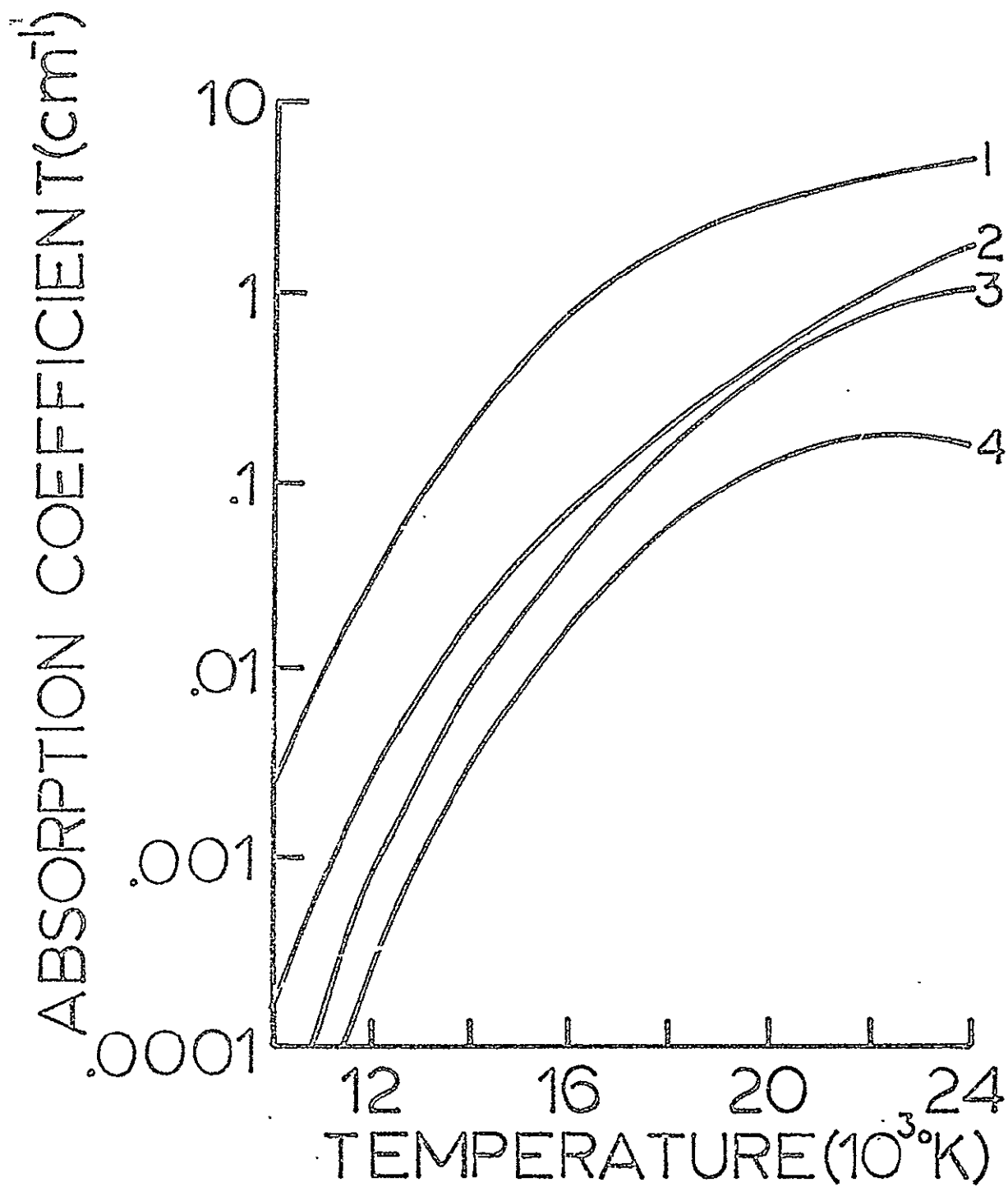
resonance absorption (which increases with increasing temperature).

Figure 20 shows our results when the absorption coefficient is computed as a function of temperature at fixed electron densities. Except at very low electron densities (where the validity of an LTE assumption becomes marginal), the absorption coefficient is not very sensitive to temperature in this case. In addition, the absorption coefficient is a multi-valued function of temperature throughout the computed range. This result is markedly different from those in Ref. 32 where no dependence of the absorption coefficient on electron density was obtained.

We, therefore, also calculated the dependence of the absorption coefficient on the electron density at fixed temperatures Here, K is very sensitive to temperature. However, the behavior becomes complicated, with increasing temperature, by a plateau regime in which a large change in the electron density can leave the absorption coefficient relatively unaffected. (See Figure 21.) Notice that the dependence of the absorption coefficient on electron density is reliably monotonic, in any event, when $n_e > 10^{18}$ cm^{-3}.

D. Conclusions

Of course, the range of temperatures within which accurate absorption measurements can be made will depend on the absorption length. However, we have found that neglecting free-free and bound-free absorption at high electron densities leads to large errors in the theoretical estimates of the relationship between absorption and temperature, viz., the serpentine trends shown in Figures 19 and 20 are missed. In addition, neglecting the effects of Doppler broadening leads incorrectly to the conclusion that
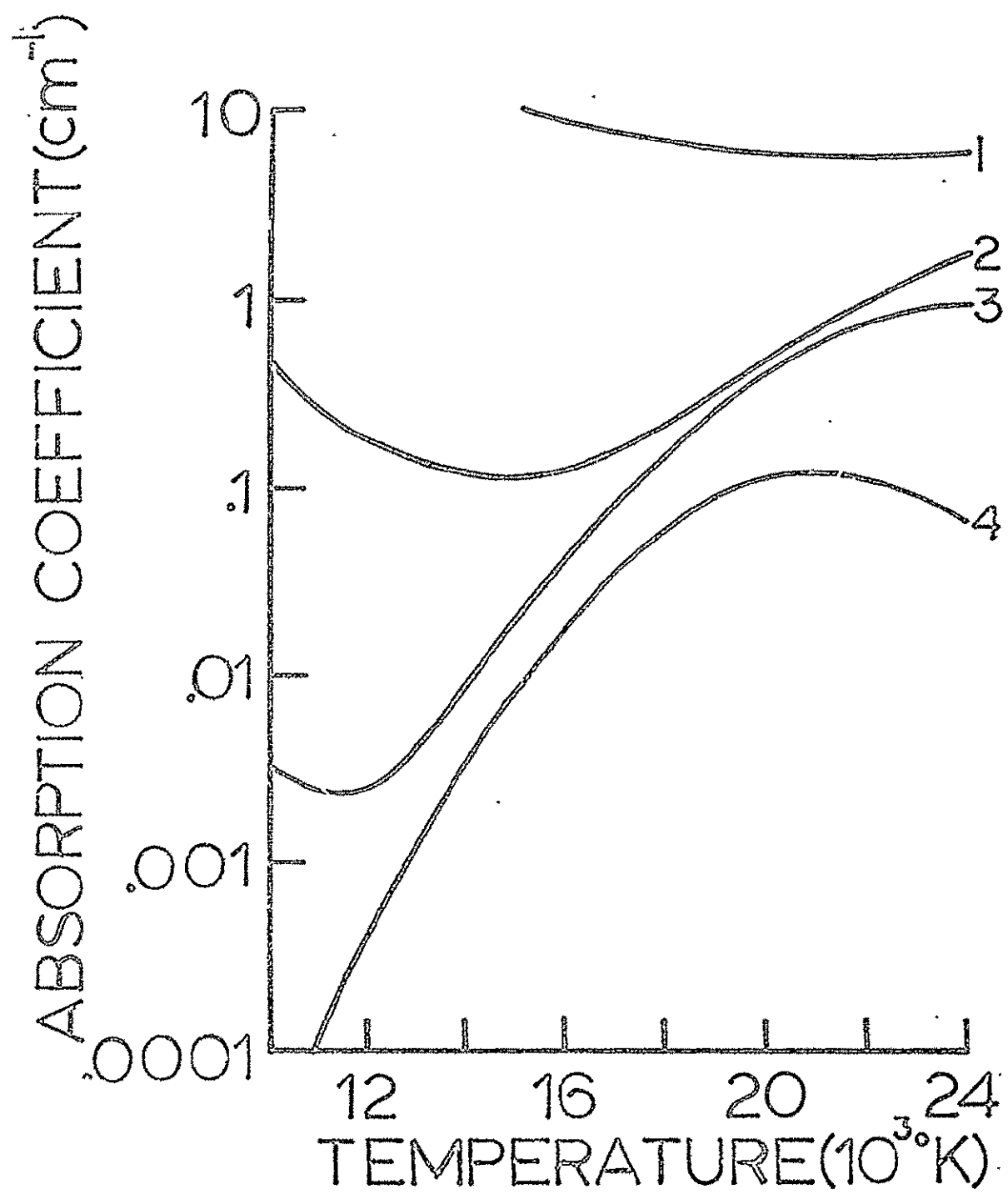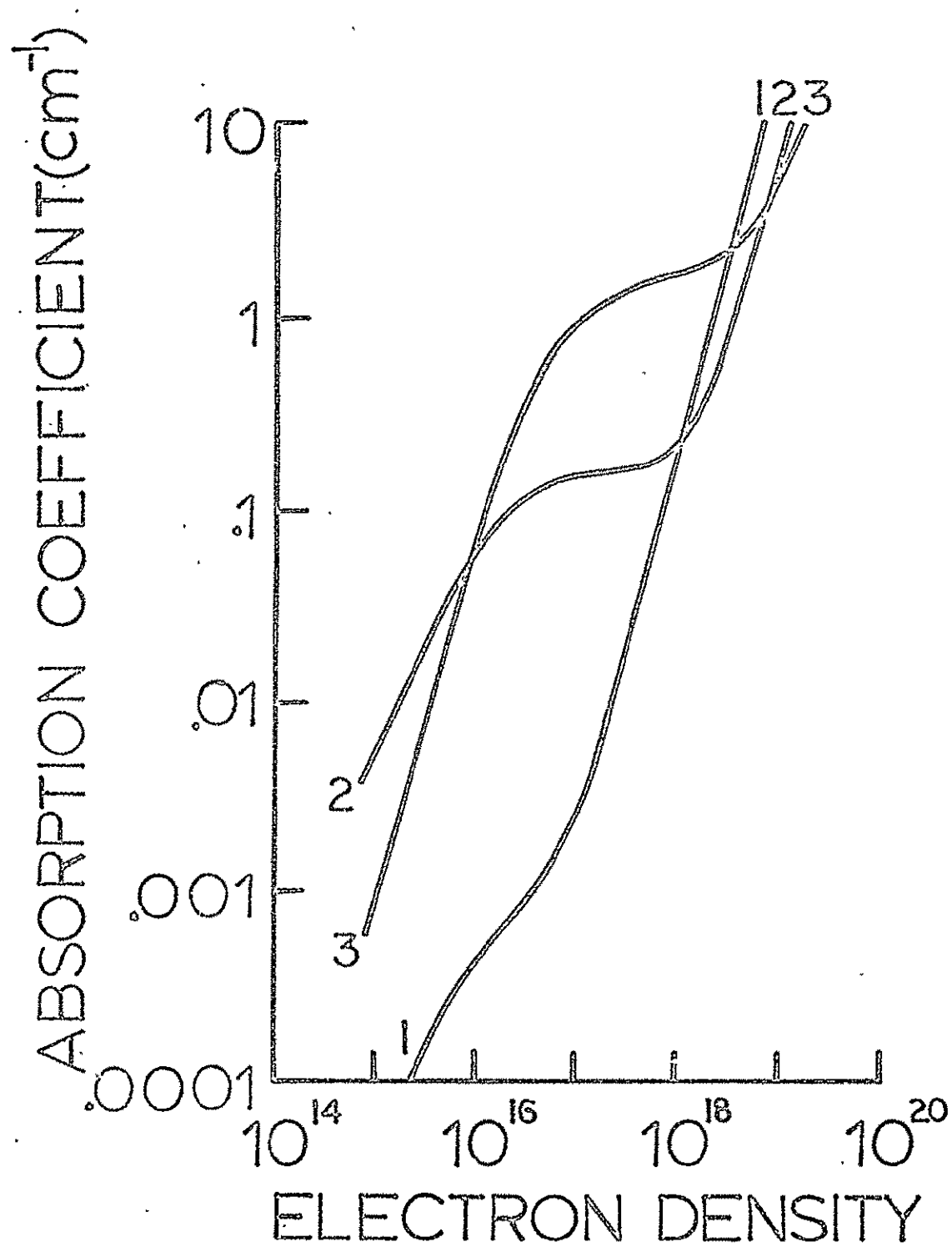
the absorption coefficient is insensitive to electron density. Therefore, the earlier suggestion[32] that absorption be used to obtain temperature measurements rests on an inappropriately oversimplified theoretical approach. From our results, it seems that absorption of 4880 $\overset{o}{A}$ radiation by an argon plasma (once the temperature is determined independently) is more suited as an electron density diagnostic.

VII. Summary

Tests for the existence and dynamic features of turbulent collisional shock fronts have not yet been performed. However, systematic trends in the performance parameters of pressure loaded arc driven shock tubes have been determined. The initial driver gas pressure is relatively unimportant. Adjusting the driver colume and the length of the discharge path can produce mixed results. Nonetheless, under the specified conditions, the Mach number increases with the cube root of capacitor energy without apparent limit. The relevance of fluorescence to ion density measurements in the ionizing shock wave has been confirmed. The absorption coefficient of an equilibrium argon plasma is computed, avoiding earlier restrictive assumptions, as a function of temperature and electron density. Our results suggest that the diagnostic applications of resonant absorption to temperature estimates are more difficult than previously expected.

When Tsuge's kinetic theory of turbulence is applied to flow with a simple nonequilibrium process, we find that the reaction rates for the turbulent and nonturbulent cases will be apparently different in direct proportion to the degree of chaos violation. Our studies of $2NO_2 + N_2 \rightleftharpoons N_2O_4 + N_2$ in an adiabatically expanding shock tube's driver gas have revealed a systematic Reynolds number based association between pressure anamolies and distortions in the NO_2 density history. We have analyzed these results using Tsuge's theory with its

boundary layer approximation. This approach seems to provide us with a nonempirical explanation of the well-known "hot spots" in turbulent nonequilibrium systems.

A boundary layer model allows us to determine characteristic scales and Reynolds numbers for a variety of turbulent shock tube discontinuities. With this model, Tsuge's kinetic theory for boundary layer flow with transition to turbulence becomes applicable. Minimum fluctuation levels as well as characteristic frequencies can be determined for unstable boundary layers with turbulent bursts. Previously published results on detonation waves have been successfully treated with these techniques. In addition, we find that our shock tube's driver gas flow produces fluctuations which can be successfully interpreted as turbulent bursts in contact surface flow. Therefore, our data and analyses support the approach to the theory of turbulence in Tsuge's works and, in particular, suggest a possible molecular interpretation for large coherent structures.

References

1. Tsuge, S., Phys. of Fluids 17 (1974) 22.

2. Klimontovich, Y.L., The Statistical Theory of Nonequilibrium Processes (M.I.T. Press, Cambridge, Mass., 1967), p. 49.

3. Tsuge, S., Sagara, K., Physics of Fluids 19 (1976) 1478.

4. Landau, L.D. and Lifshitz, E.M., Fluid Mechanics, p. 116, Addison-Wesley Publ. Co., Reading, Mass., 1959.

5. Hall, J.G., J. Appl. Phys. 26, 698 (1955).

6. Istratov, A.G., Korovjanskaja, E.I., and Librovich, V.B., 12th Symposium (International) on Combustion, p. 779, Academic Press, New York, 1969.

7. Tsuge, S., Phys. Fluids 17, 22 (1974).

8. Osborne, M., S.I.A.M. J. Appl. Math. 15, 539 (1967).

9. Jordinson, R., J. Fluid Mech. 43, 801 (1970).

10. Tsuge, S. and Sagara, K., Phys. Fluids 19, 1478, (1976).

11. Johnson, J.A., III and Chen, S.C., Phys. Letts. 68A, 141 (1978).

12. White, D.R., Phys. Fluids 4, 465 (1961).

13. Lien, H. and Eckerman, J., A.I.A.A. J. 4, 1988 (1966).

14. Tsuge, S., Phys. Fluids, 17 22-23 (1974).

15. Tsuge, S. and Sagara, K., J. Stat. Phys., 12 403-25 (1975).

16. Tsuge, S. and Sagara, K., Phys. Fluids 19 1478-85 (1976).

17. Johnson, J.A., III and Chen, S.C., Phys. Letts. 68 141-3 (1978).

18. Landau, L. and Lifshitz, E., Fluid Mechanics (Reading, Mass.: Addison-Wesley) pp. 114-6 (1959).

19. Hall, J.G., J. Appl. Phys. <u>26</u> 698-700 (1955).

20. Thompson, P.A., Compressible-Fluid Dynamics (New York: McGraw-Hill) pp. 374-442 (1972).

21. White, D.R., Phys. Fluids <u>4</u> 465-80 (1961).

22. Bradley, J.N., Shock Waves in Chemistry and Physics (London: Methuen) pp. 135-47 (1962).

23. Zimet, E., J. Chem. Phys. <u>53</u> 515 (1970).

24. Tennekes, H. and Lumley, J.L., A First Course in Turbulence (London: M.I.T. Press) pp. 210-1 (1972).

25. Dannenberg, R.E. and Silva, A.F., A.I.A.A. J. <u>10</u>, 1563. (1972).

26. Dannenberg, R.E., A.I.A.A. J. <u>10</u>, 1692 (1972).

27. Menard, W.A., A.I.A.A. J. <u>9</u>, 2096 (1971).

28. Warren, W.R. and Harris, C.J., Aerospace Corporation Report (Sept., 1969) (Contract No. F04701-69-C-0066).

29. Dannenberg, R.E., Proceedings of the 11th International Symposium on Shock Tubes and Waves (1977). (To be published.)

30. Bradley, J.N., <u>Shock Waves in Physics and Chemistry</u>, Wiley Pub. Co. (1962).

31. Smy, P.R., Nature <u>193</u>, 969 (1962).

32. Wheeler, C.B. and Troughton, J., Plasma Phys. <u>11</u>, 391 (1969).

33. Lochte-Holtgreven, W., <u>Plasma Diagnostics</u> (North-Holland, Amsterdam, 1968) pp. 40-41, pp. 35-36.

34. Schnqpauff, R., Z. Astrophys. <u>68</u>, 431 (1968).

35. Zel'dovich, Ya. B., and Raizer, Yu. P., <u>Physics of Shock Waves and High Temperature Hydrodynamic Phenomena</u> (Academic Press, New York, 1967) Vol. I, pp. 281-283.

36. Engelhardt, A.G., and Phelps, A.V., Phys. Rev. 133, A375
    (1964).

37. See, for example, Yu. P. Raizer, Sov. Phys. Uspekhi 8, 650
    (1966).

38. Schluter, D., Z. Phys. 210, 80 (1968).

39. Vorypaeva, N.S., Nikolaevskii, L.S. and Podomoshenskii, I.V.,
    Opt. Spectrosc. (USSR) 44, 247 (1978).

40. Labat, J., Djenize, S., Cirkovic, Lj and Puric, J., J. Phys.
    B: Atom Molec. Phys. 7, 1174 (1974).

41. Griem, H., Plasma Spectroscopy (McGraw-Hill, New York, 1964)
    pp. 150-159.

42. Moore, C.E., Circular of the National Bureau of Standards
    467 (U.S. Govt. Printing Office, Washington, D.C., 1949).

APPENDIX A

BIBLIOGRAPHY: NSG 1314

"Velocity Relaxation in Shock-Heated Fully Turbulent Gases",
    Bull. Am. Phys. Soc., 22, 1253 (1977). (J.A. Johnson,
    K. Oyedeji and S. Chen.)

"Fluctuation Agglomerates in Transitional Flow", Bull. Am. Phys.
    Soc., 22, 1280 (1977). (J.A. Johnson)

"Some Evidence of Reaction Distortion in Turbulent Nonequilibrium
    Flow", Bull. Am. Phy. Soc., 24, 75 (1979). (J.A. Johnson,
    R. Ramaiah and J. Santiago.)

"A New Approach to Statistical Fluid Mechanics", Bull. Am. Phys.
    Soc., 24, 460 (1979). (J.A. Johnson)

"Concerning a Molecular Basis for Understanding Transition to
    Turbulence in Gasdynamic Discontinuities", Bull. Am. Phys.
    Soc., 24, 460 (1979). (J.A. Johnson and Lin I.)

"A Molecular Interpretation of Large Coherent Structures in
    Turbulent Flow", Bull. Am. Phys. Soc., (1979, to be published).
    (J.A. Johnson, R. Ramaiah and Y. Li).

"Pressure Bursts and Product Enhancement in Fluctuating Reacting
    Flow", Bull. Am. Phys. Soc., (1979, to be published).
    (J.A. Johnson and R. Ramaiah)

"Strong Shock Waves from a Low Capacity Discharge", Bull. Am.
    Phys. Soc., (1979, to be published). (J.A. Johnson, Lin I
    and J. Santiago)

"The Absorption Coefficient of an Equilibrium Argon Plasma at 4880 Å'
    Bull. Am. Phys. Soc., (1979, to be published). (J.A. Johnson
    and J. Santiago)

"Reaction Rate Distortion in Turbulent Flow", Phys. Letts., 68A,
    141 (1978). (J.A. Johnson and S.C. Chen)

"Improving the Mach Number Capabilities of Arc Driven Shock Tubes",
    (Preprint, 1978, submitted for publication). (J.A. Johnson
    and J. Santiago)

"Turbulent Bursts in a Shock Tube", (Preprint, 1979, submitted for
    publication). (J.A. Johnson)

"Resonant Absorption in an Argon Plasma at Thermal Equilibrium",
    (Preprint, 1979, submitted for publication). (J.A. Johnson
    and J. Santiago)

"Driver Gas Flow with Fluctuations", (Preprint, 1979, submitted for
    publication). (J.A. Johnson, W. Jones and J. Santiago)